General Bit Error Ratio Analysis of Interference Affected M-PSK Transmissions

(Invited Paper)

László Pap, *Senior Member, IEEE,* Albert Mráz, *Member, IEEE*

***Abstract*—The authors of current work provide a performance analysis in terms of the bit-error-ratio of gray-mapped M-PSK transmission. The analysis results exact and closed form expressions which consider the effects of different interference scenarios with arbitrary number of interference sources and multiple types of fading scenarios on the propagation of the analyzed signal. Both effects have been modeled stochastically.**

***Index Terms*—M-PSK, BER, Rayleigh-, Rice-, Nakagami-fading, stochastic interference model.**

I. INTRODUCTION

EXACT and closed form bit-error-ratio (BER) calculation method will be provided in this paper for M-ary phase-shift keying (PSK) transmission, considering arbitrary number of interference sources and different fading models affecting the propagation of the useful signal.

Numerous works are present in the literature, which deal with solutions for the error analysis of M-PSK transmission. Paper [1] – referring the related existing calculation methods – contains an exact symbol-error-ratio (SER) calculation for M-ary quadrature modulation (QAM) and M-PSK transmission, which contains an extended interference- and fading model for the calculation, generally defined in [2]. The referred model contains a universal fading calculation method and interference model for M level frequency shift keying (FSK) with the *fully separated* handling of the interference and fading effects, resulting a gainful and extendable tool for considering almost arbitrary fading- (Rayleigh, Rice and Nakagami) and interference situations. The contribution of [1] means a novelty for the interference model by considering the effects of the spectral overlap between the useful- and interfering signals, calculating with the spectral shapes and the frequency-domain distances of them. The model has been extended to arbitrary number of OFDM interferers with sinc spectral shapes.

A previous work of current authors [3] provides an extension of [1] towards the BER calculation of M-QAM over the previously considered fading channels and involving the extended interference model of [1]. The developed solution provides an extension for [4] as an exact BER calculation of the gray-mapped M-QAM. However it deals only with additive white Gaussian-noise (AWGN) and without any interference effects. The contribution of [3] expands the BER calculation with interference- and fading models of [2] with some transformations in reference to the original BER expression to make it able to involve the defined models.

László Pap and Albert Mráz are with the Department of Telecommunications, Budapest University of Technology and Economics, Budapest H-1117, Hungary (e-mail: pap@hit.bme.hu, mraz@hit.bme.hu).

Manuscript received August 1, 2011; revised August ?, 2011.

This paper aims to provide exact closed form BER calculation for gray-mapped M-PSK transmission with different newly defined interference models and for the extension of the spectral overlap calculation of [1]. Within this area a SER calculation has already been interpreted in [5] containing the effects of imperfections during the synthesis of the in-phase- and quadrature components within the M-PSK transmitter. It was expressed for AWGN channel with the help of the two-dimensional Gaussian error functions. The SER calculation of M-PSK has been extended to BER calculation with gray mapping and fading channels affecting the useful signal [6]. Nevertheless, the random properties of the interference have not been investigated. An other path for the calculation of the BER of M-PSK has been given in [7] presenting exact closed form BER expression for 8-PSK. However for larger M values only (tight) upper and lower bounds have been given. The authors of [8] have provided an improvement for [7] resulting an exact solution for *arbitrary* M levels for AWGN and gray mapping likewise.

This paper has the following structure. In Section II a framework will be proposed for the calculation of the BER of M-PSK according to [8] valid for AWGN and gray mapping, which will be extended to the mathematical form defined in [2] and [1] with the investigation of the spectral overlap among the interferers and introducing an extended interference model within Sections III and IV. A transformed form of the M-PSK BER expression will be given in Section V, which is extendable to consider various interference- and fading models. Three different interference scenarios will be analyzed in Section VI and illustrated with calculation results.

II. BIT ERROR RATE OF M-PSK IN AWGN CHANNEL

A solution has been provided in [8] for the calculation of the average BER of M-PSK for AWGN and assuming gray coding. The solution has been a refinement of the calculation method in [7], which gives only an upper bound for $M \geq 16$, where M denotes the size of the modulation alphabet. The reason of this inaccuracy has been shown in [8] and the following expression has been resulted for the average BER of gray-coded M-PSK

$$P_b = \frac{1}{\log_2(M)} \sum_{m=1}^{M-1} \overline{d}\,(m)\, P\,(m), \tag{1}$$

in which $\overline{d}\,(m)$ represents a so-called average distance spectrum, denoting the average number of bit positions differing adequately to the alternatives of the received signal with the

description of

$$\overline{d}(m) = 2\left|\frac{m}{M} - \left\lfloor\frac{m}{M}\right\rceil\right| + 2\sum_{i=2}^{\log_2(M)}\left|\frac{m}{2^i} - \left\lfloor\frac{m}{2^i}\right\rceil\right|, \quad (2)$$

where $\lfloor . \rceil$ denotes the rounding to the nearest integer. Within expression (1) $P(m)$ represents the probability, that the received signal vector falls into the decision region of R_m in the case of $X = 0$ has been sent as

$$P(m) = \Pr\{\mathbf{Y} \in R_m | X = 0\}, \quad (3)$$

where X denotes the transmitted symbol with $X \in \{0, 1, ..., M-1\}$. The expression of $P(m)$ has been defined and calculated in [7, eq. (3)] (denoted by A_m) as

$$P(m) = \int_0^\infty f(z)\left[Q\left[\vartheta + z\cdot\tan\left(\left(\frac{M}{2} - 2m - 1\right)\frac{\pi}{M}\right)\right] - \right.$$
$$\left. -Q\left[\vartheta + z\cdot\tan\left(\left(\frac{M}{2} - 2m + 1\right)\frac{\pi}{M}\right)\right]\right]dz, \quad (4)$$

in which $\vartheta = \sqrt{2\cdot\log_2(M)\frac{E_b}{N_0}}$ with E_b being the bit-energy and N_0 the spectral power density of the AWGN, $f(z) = \exp\left(-z^2/2\right)/\sqrt{2\pi}$, while $Q(x) = 1/\sqrt{2\pi}\int_x^\infty \exp\left(-u^2/2\right)du$ represents the tail probability of the standard normal distribution. Note, we have performed some modifications in terms of the notations compared to [7] and [8] to avoid the handling of unnecessary variables.

In Section V we will extend the BER expression defined in (1) – valid for AWGN channel – in a similar way to [1] and [3], in order to consider a general interference model. However, this extension could only be performed if $P(m)$ would have a special *exponential integral* form, which is not true for (4). Furthermore, the $Q(.)$ components of it cannot be simply transformed into the desired form. It means that an alternate method needs to be invented to calculate $P(m)$. Towards this, let consider the constellation model of Fig. 1. The streaked part of the circle represents the decision region R_m. With the notations of the model

$$\tan(\varphi_1(m)) = \frac{\mathbf{r}_1\sin(\theta)}{\sqrt{E_s} - r_1\cos(\theta)},$$

from which the $\mathbf{r}_1$ distance vector (to $X = 0$) can be expressed as

$$\mathbf{r}_1 = \sqrt{E_s}\frac{\tan(\varphi_1(m))}{\sin(\theta) + \tan(\varphi_1(m))\cos(\theta)},$$

with $\varphi_1(m) = \pi\frac{2m-1}{M}$ and $\varphi_2(m) = \pi\frac{2m+1}{M}$. After that, the $P(m)$ probability can be expressed with a surface integral for

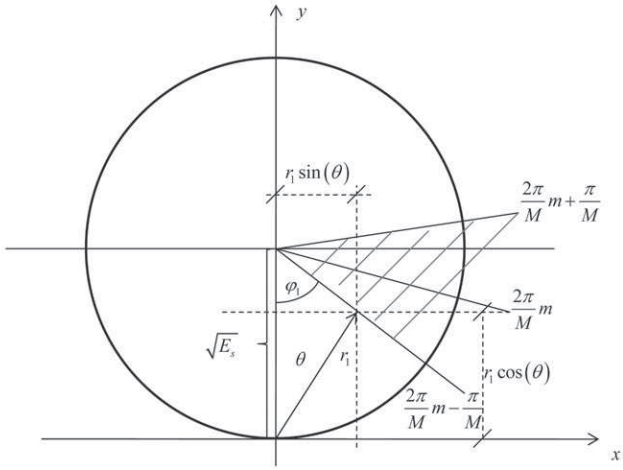

Fig. 1: Model to calculate $P(m)$ probability with the help of the polar coordinates

the signal space $\mathcal{A}$ as

$$P(m) = \iint_{\mathcal{A}}\frac{1}{\pi N_0}\exp\left(\frac{\mathbf{r}_1^2}{N_0}\right)drd\theta =$$
$$= \frac{1}{2\pi}\int_0^{\beta_1(m)}\exp\left(-\frac{E_s}{N_0}\frac{\sin^2(\varphi_1(m))}{\sin^2(\varphi_1(m)+\theta)}\right)d\theta - \quad (5)$$
$$-\frac{1}{2\pi}\int_0^{\beta_2(m)}\exp\left(-\frac{E_s}{N_0}\frac{\sin^2(\varphi_2(m))}{\sin^2(\varphi_2(m)+\theta)}\right)d\theta,$$

containing the E_s symbol energy and the $\beta_1(m) = \pi - (2m-1)\pi/M$ and $\beta_2(m) = \pi - (2m+1)\pi/M$ integration limits. Following the transformation above, we will be able to re-calculate the results of [8]. At the same time, we have originally aimed for making make the expression of P_b suitable to be an object of a further extension after which almost-arbitrary interference scenarios could be modeled during the calculations.

III. CALCULATION OF THE SPECTRAL OVERLAP

It was shown in [1] that the frequency-domain distance and the spectral shapes of the useful- and interfering signals influences the final BER results. To model this effect, a so-called *spectral overlap* factor has been defined as

$$v(\Delta f_k) = \frac{\int_{-\infty}^{\infty} S_{\eta_k}(f - \Delta f_k)\,S(f)\,df}{\int_{-\infty}^{\infty} S(f)\,df}, \quad (6)$$

where $S_{\eta_k}(f)$ was defined the spectral shape of interferer k with the dimension of $[\frac{1}{\text{Hz}}]$ and an $\eta_k(t)$ complex Gaussian process has been introduced with independent real- and imaginary components and $R_{\eta_k}(0) = 1$. $S(f)$ will be considered as the spectral power density of the useful signal in $[\frac{\text{W}}{\text{Hz}}]$. After that, the amount of $v(\Delta f_k)$ is expected to be a value with the dimension of [s]= $[\frac{1}{\text{Hz}}]$, which is proportional to the spectral shapes of the useful- and interfering signals and the s_0 power of the useful signal.

A. Interferers with sinc spectral shape

Expression (6) was calculated in [1] for OFDM interference. In this case the $S_{\text{OFDM}}(f)$ spectral power density function of an OFDM subcarrier has been assumed to have a pulse shape for the $s(t)$ time-domain signal in terms of

$$s(t) = \begin{cases} \sqrt{s_0}\ [\text{V}],\ \text{if } t \in \left[-\frac{T_\text{s}}{2}, \frac{T_\text{s}}{2}\right) \\ 0,\ \text{otherwise,} \end{cases}$$

where T_s represents the symbol period. Thus the spectral power density function of $s(t)$ after applying Fourier-transform can be expressed as

$$S_{\text{OFDM}}(f) = \\ = \frac{s_0}{T_\text{s}} \left| \int_{-\frac{T_\text{s}}{2}}^{\frac{T_\text{s}}{2}} e^{-j2\pi f t} dt \right|^2 = s_0 T_\text{s} \left(\frac{\sin(\pi f T_\text{s})}{\pi f T_\text{s}} \right)^2 \left[\frac{\text{W}}{\text{Hz}}\right] \quad (7)$$

as a variant of the $\text{sinc}(x) = \sin(x)/x$ function. Similarly, assuming the same T_s symbol period for the interferers, the spectral power density function of interferer k can be given with $S_{\eta_k,\text{OFDM}}(f) = T_\text{s}\left(\frac{\sin(\pi f T_\text{s})}{\pi f T_\text{s}}\right)^2$. It was shown in [1] that for $l \in \mathbb{Z}$

$$v_{\text{OFDM}}(\Delta f_k) = v_{\text{OFDM}}(l \cdot \Delta f_\text{c}) = \frac{T_\text{s}}{l^2 \pi^2} \left[\frac{1}{\text{Hz}}\right], \quad (8)$$

in which $\Delta f_k = |f_0 - f_k|$ represents the frequency domain distance of the f_k center frequency of interferer k from the f_0 carrier frequency of the useful signal. The expression above means that for Δf_k values, which can be expressed with the integer l multiples of the Δf_c subcarrier spacing, the calculation of $v_{\text{OFDM}}(.)$ represents a reasonably simpler task.

B. Interferers with raised cosine spectral shape

The calculation of the spectral overlap above is valid for OFDM transmission. Nevertheless, PSK modulation is not frequently applied for OFDM transmission. To create a realistic interference model, let consider spectral shapes generated by a raised cosine (RC) filter. For assuming a simplified form of the $|G_\text{rc}(f)|$ spectral shape of a signal generated by a RC filter, let consider a *unit roll-off factor* [9] for that. After that

$$|G_\text{rc}(f)| = \begin{cases} \frac{T_\text{s}}{2}\left[1 + \cos(\pi f T_\text{s})\right] \left[\frac{1}{\text{Hz}}\right], \text{if } |f| \le \frac{1}{T_s} \\ 0, \text{ otherwise} \end{cases} \quad (9)$$

We know that $|G_\text{rc}(f)|^2 = \frac{T_s}{s_0} S_\text{rc}(f)$ with the $S_\text{rc}(f)$ spectral power density of the useful signal. During the calculation of the integral in (6) applied to the current case, the numerator will be expressed as $\int_{-1/T_\text{s}+\Delta f_k}^{1/T_\text{s}} S_{\eta_k,\text{rc}}(f - \Delta f_k) S_\text{rc}(f)\, df$, with $S_{\eta_k,\text{rc}}(f) = \frac{1}{T_s}|G_\text{rc}(f)|^2$. For the denominator it can be shown, that $\int_{-\infty}^{\infty} S_\text{rc}(f)\, df = \frac{3}{4} s_0$. Then the spectral overlap

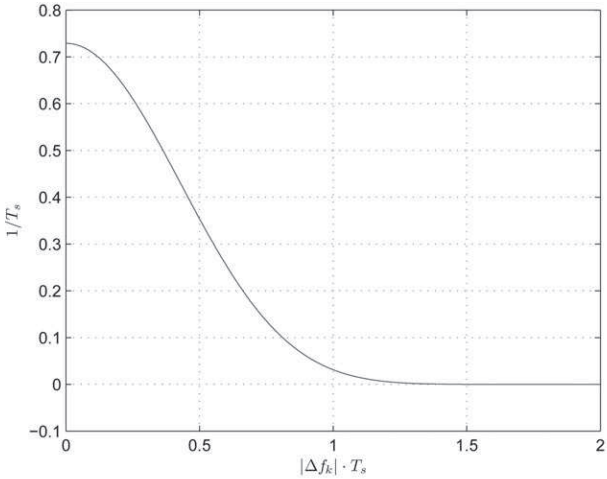

Fig. 2: Illustration of the $v_\text{rc}(\Delta f_k)$ overlap for raised cosine spectral shapes of the interferers and useful signal

can be given with

$$v_\text{rc}(\Delta f_k) = \frac{3T_s}{8} - \frac{3\Delta f_k T_s^2}{16} + \\ + \frac{T_s}{6}(\Delta f_k T_s - 2)\cos(\pi \Delta f_k T_s) + \\ + \frac{T_s}{96}(\Delta f_k T_s - 2)\cos(2\pi \Delta f_k T_s) + \frac{5T_s}{18\pi}\sin(\pi \Delta f_k T_s) + \\ + \frac{25T_s}{576\pi}\sin(2\pi \Delta f_k T_s) \quad (10)$$

with the dimension of $1/[\text{Hz}]$ for $0 < |\Delta f_k| \le \frac{2}{T_s}$, since the signal contains any power only within this interval. Fig. 2 contains the illustration of $v_\text{rc}(\Delta f_k)$. It is visible that $v_\text{rc}(\Delta f_k) = 0$ for $|\Delta f_k| \ge \frac{2}{T_\text{s}}$, i.e. no interference effect will be resulted with larger freq. distances than $\frac{2}{T_\text{s}}$ in contrast to the OFDM case with $v_{\text{OFDM}}(\Delta f_k) > 0$ to arbitrary large Δf_k values at the same time.

IV. EXTENDED SNR MODEL

Following the introduction of the spectral overlap concept, we return to the insertion of the interference – as it was defined in [1] – into a common 'signal-to-noise model' of the BER calculation, which generally considers the $\frac{E_\text{b}}{N_0}$ fraction. An effective γ signal-to-noise ratio has been introduced, which contains an extended N_0' amount considering the effects of the spectral power density of the received 'original' N_0 additive white Gaussian noise and the effective interference energy. See details in [1, Eq. (14)]. The extended signal-to-noise ratio has been expressed as

$$\gamma = \frac{E_b}{N_0'} = \frac{s_0 T_b}{N_0'} = \frac{s_0 T_b}{N_0 + \sum_{k=1}^{K} s_k \cdot v(\Delta f_k)}, \quad (11)$$

in which T_b denotes the bit time-interval and s_k represents the instantaneous received power of interferer k with $k \in (1, 2, ..., K)$, where K represents the total number of

interferers. In addition $\sum_{k=1}^{K} s_k \cdot v(\Delta f_k)$ also contains the $v(\Delta f_k)$ spectral overlap formerly expressed in equations (8) and (10) and the reason why it can be handled additively to N_0 is that the interference is assumed to propagate across a complex Gaussian (Rayleigh fading channel) characterized by the previously defined $\eta_k(t)$ process.

V. EXTENSION OF THE BER CALCULATION

After considering the interference as an additive quantity to the AWGN, we will be be able to deal with different interference models. Nevertheless, we should consider and solve some computational problems assuming some parameters as *random variables*. This computational solution will be detailed within the current section.

A. Insertion of the interference model

Let us denote the elements of expression (5) with $f_1(s_0, m)$ and $f_2(s_0, m)$ according to

$$P(m) = f_1(s_0, m) - f_2(s_0, m)$$

which will be the functions of s_0 and m. Furthermore, $C_1(\theta, m) = \frac{\sin^2(\varphi_1(m)+\theta)}{\sin^2(\varphi_1(m))}$ and $C_2(\theta, m) = \frac{\sin^2(\varphi_2(m)+\theta)}{\sin^2(\varphi_2(m))}$, while $E_b = s_0 T_b$ with $T_b = \frac{T_s}{\log_2(M)}$. Let us substitute the extended (containing interference) γ effective signal-to-noise-ratio value defined in (11) into the components of $P(m)$ in expression (5), yielding

$$f_1(s_0, m|\mathbf{s}, \mathbf{f}) = \\ = \frac{1}{2\pi} \int_0^{\beta_1(m)} \exp\left(-\frac{E_s}{N_0 + \sum_{k=1}^{K} s_k \cdot v(\Delta f_k)} \frac{1}{C_1(\theta, m)} \right) d\theta \tag{12}$$

and

$$f_2(s_0, m|\mathbf{s}, \mathbf{f}) = \\ = \frac{1}{2\pi} \int_0^{\beta_2(m)} \exp\left(-\frac{E_s}{N_0 + \sum_{k=1}^{K} s_k \cdot v(\Delta f_k)} \frac{1}{C_2(\theta, m)} \right) d\theta, \tag{13}$$

where $\mathbf{s} = (s_1, s_2, ..., s_K)$ and $\mathbf{f} = (f_1, f_2, ..., f_K)$ represent the vectors constituted of the instantaneous received power levels and the center frequencies of the interference sources respectively.

As it has been emphasized both in [2] and [3], expressions (12) and (13) are conditional quantities with two sets of conditions $\mathbf{s}$ and $\mathbf{f}$, since the elements of these two vectors are considered as *random variables* (RVs). To get the unconditional forms of $f_1(s_0, m)$ and $f_2(s_0, m)$, we should execute an expected value calculation with respect to the many RVs defined above. It has been already discussed, that these calculations would require a K-fold integration severally to (12) and (13) in terms of the s_k and f_k RVs. In addition $\sum_{k=1}^{K} s_k \cdot v(\Delta f_k)$ is a sum of RVs and it is located *within the denominator* of γ, which leads to further hardness regarding the calculation. Furthermore, s_0 can be also considered as a RV. To solve this difficult computational challenge, the authors of [2] have introduced a transformation according to the following lemma.

Lemma 1: For any $x, y > 0$

$$\exp\left(-\frac{y}{x}\right) = 1 - \int_0^{\infty} \frac{J_1(2\sqrt{z})}{\sqrt{z}} \exp\left(-z\frac{x}{y}\right) dz \tag{14}$$

where $J_1(.)$ represents the Bessel function of first order and first kind.

Applying (14), the numerators and denominators of expressions (12) and (13) will be reversed as

$$f_1(s_0, m|\mathbf{s}, \mathbf{f}) = \\ = \frac{1}{2\pi} \int_0^{\beta_1(m)} \left[1 - \int_0^{\infty} \frac{J_1(2\sqrt{z})}{\sqrt{z}} \exp\left(-z\frac{N_0}{s_0 T_s} C_1(\theta, m) \right) \cdot \right. \\ \left. \cdot \exp\left(-\frac{z}{s_0 T_s} \sum_{k=1}^{K} s_k \cdot v(\Delta f_k) C_1(\theta, m) \right) dz \right] d\theta \tag{15}$$

and

$$f_2(s_0, m|\mathbf{s}, \mathbf{f}) = \\ = \frac{1}{2\pi} \int_0^{\beta_2(m)} \left[1 - \int_0^{\infty} \frac{J_1(2\sqrt{z})}{\sqrt{z}} \exp\left(-z\frac{N_0}{s_0 T_s} C_2(\theta, m) \right) \cdot \right. \\ \left. \cdot \exp\left(-\frac{z}{s_0 T_s} \sum_{k=1}^{K} s_k \cdot v(\Delta f_k) C_2(\theta, m) \right) dz \right] d\theta. \tag{16}$$

Let us consider the benefits of this seemingly difficult transformation. The significant advantage of (14) is that the s_0 instantaneous power of the useful signal and the $\sum_{k=1}^{K} s_k \cdot v(\Delta f_k)$ interference become able to be *handled independently* during the calculation, and the necessity of the K-fold integrations could be eliminated, since the expected value will be able to calculated with the help of the *moment generating function*. The detailed modeling of the different interference scenarios will be demonstrated in the following section.

VI. EXAMPLES OF THE INTERFERENCE MODEL

Based on the transformation in Section V, the sum of the several random variables will appear in the exponent within (15) and (16). This fact will enable us to calculate the expected value in terms of their moment generating function. Let us express the expected value of the interference-related exponential component of $f_1(s_0, m|\mathbf{s}, \mathbf{f})$ defined in (15) with

$$\phi_1(z, \theta, m) = \mathbb{E}\left[e^{-\frac{z}{s_0 T_s} \sum_{k=1}^{K} s_k v(\Delta f_k) C_1(\theta, m)} \right]. \tag{17}$$

Note that $\phi_2(z, \theta, m)$ can be also calculated similarly with the help of $C_2(\theta, m)$. In the following we will only consider the calculation of $\phi_1(z, \theta, m)$ due to the high level similarity.

The s_k and f_k RVs can be also assumed independent with identical distribution (i.i.d.) for each k. In this case the sum

within the exponent will be eliminated, and we will get a more simple form with

$$\phi_1(z,\theta,m) = \mathbb{E}\left[e^{-z\frac{s_k}{s_0 T_s} v(\Delta f_k) C_1(\theta,m)}\right]^K. \qquad (18)$$

In the following we will introduce and calculate $\phi_1(z,\theta,m)$ for three different interference scenarios.

A. Frequency hopping with OFDM interference

Let us define a scenario with 'OFDM-like' interference, meaning that 'sinc' spectral shapes (see in Section III-A) will be considered. We calculate the expected value of (18) with respect to the frequency distances of interferers to the f_0 center frequency of the useful signal. As we have seen in (8), the $v_{\text{OFDM}}(\Delta f_k)$ depends on the $l_k \in \mathbb{Z}$ parameters in a very simple way. It this interference model l_k will be considered as RVs, hence the f_k center frequencies of the interferers will be located with freq. distances expressed as *integer multiples* of a Δf_c subcarrier spacing from f_0 according to $\Delta f_k = l_k \Delta f_c = |f_0 - f_k|$. The f_k can be located at both sides of f_0, since Δf_k represents the absolute value of the frequency domain distance. Let consider $s_k = s_1$ and $l_k = l$, $\forall k$. In this case, and with the assumption that l has a discrete uniform distribution defined by $l \in \{1, 2, ..., N\}$, the expected value defined in (18) can be calculated as

$$\begin{aligned}\phi_1(z,\theta,m|s_1) &= \mathbb{E}\left[e^{-z\frac{s_1}{s_0 T_s} v(l\cdot\Delta f_c) C_1(\theta,m)}\right]^K = \\ &= \left[\frac{1}{N}\sum_{l=1}^{N} e^{-z\frac{s_1}{s_0}\frac{1}{l^2\pi^2} C_1(\theta,m)}\right]^K = \\ &= \left[\frac{1}{N}\sum_{l=1}^{N} \exp\left(-z\frac{10^{-\frac{\alpha_{\text{dB}}}{10}}}{l^2\pi^2} C_1(\theta,m)\right)\right]^K,\end{aligned} \qquad (19)$$

where N will represent the number of possible Δf_k frequency distance values and α_{dB} denotes s_1/s_0 in [dB]. For larger N the interferers with fixed K number will be located within a wider $B_{\text{ch}} = 2N \cdot \Delta f_c = N_c \cdot \Delta f_c$ bandwidth, resulting larger Δf_k distances with more and more probability, yielding lower average interference in terms of the expression of the spectral overlap with the $N_c = 2N$ number of the f_k position possibilities. The phenomena above can be observed at Fig. 3 with the calculated BER curves for 4-, 8- and 16-PSK, with $K = 1$ and assuming Rice-fading channel for the propagation of the useful signal with parameter $\kappa = 10$. To consider the fading effects, the expected value of the $\frac{\sqrt{\xi} J_1(2\sqrt{z\xi})}{\sqrt{z}}$ component of (15) and (16) should be calculated according to the ξ instantaneous fading power gain affecting the useful signal. The calculation has been detailed in [2, eq. (48)]. During our further investigations Rice-fading will be assumed. The curves of Fig. 3 have been calculated for $N = 16$ and 128 resulting lower interference and lower BER for a larger N value. The degradation effects of the same amount of interference growth results higher BER performance fall-off with the larger M level.

Let consider the adjustment of K. At Fig. 4 the BER curves are shown for the same M levels as in the previous case with fixed $N = 16$. The number of interferers was set to $K = 1$

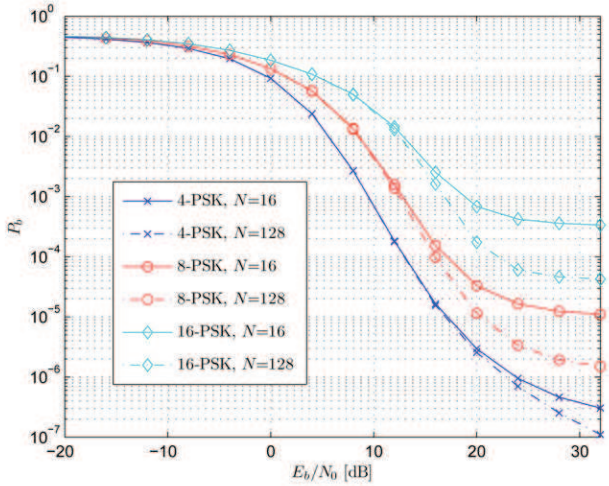

Fig. 3: BER curves for 4-, 8- and 16-PSK in case of Rice-fading with $\kappa = 10$, $\alpha_{\text{dB}} = 10$ dB and different number of possible Δf_k interferer center frequency distances N =16 and 128

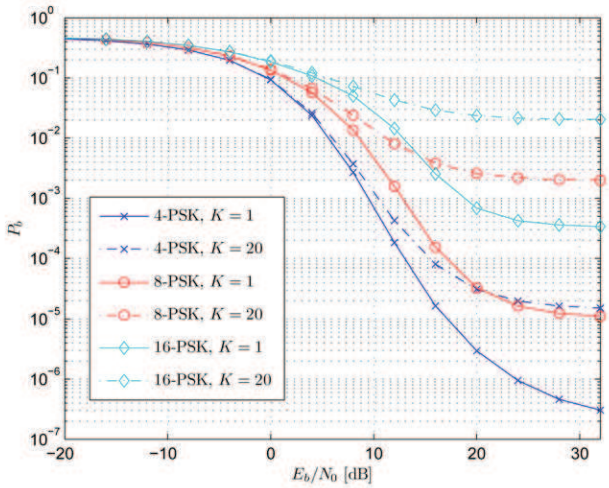

Fig. 4: BER curves for 4-, 8- and 16-PSK in case of Rice-fading with $\kappa = 10$, $\alpha_{\text{dB}} = 10$ dB and different number of interferers K=1 and 20

and 20. It can be clearly observed, that with the increased number of interferers with a fixed B_{ch} bandwidth the BER performance will also suffered in an growing manner with M.

B. Poisson field of interferers

In this interference scenario we consider a situation in which the K number of interferers in a defined area A will be considered as a Poisson RV with the average of λ [Interferer/m^2/Hz]. In this case $\Pr\{K = k\} = \frac{(A\lambda)^k}{k!} e^{-A\lambda}$. Let r_k denote the distance between the interferer k and the receiver. We calculate $s_k = s_{k,t} r_k^{-\beta}$ with β being the pathloss exponent. The distance r_k has the distribution of $f(r_k) = 2r_k/(D^2 - D_0^2)$ with the D radius of the investigated area and a D_0 initial radius from the receiver with the same center position. We have considered $s_{k,t}$ as the transmit power of interferer k for which $s_{k,t}/s_0 = 1$ will be assumed. We would like to find $\phi_1(z,\theta,m)$ by calculating

the expected value with respect to three different RVs: K, $r_1 = r_k$ for $\forall k$ and l. The variable l will denote the same quantity as in VI-A, since we will assume sinc spectral shapes for the interferers. Then we arrive at

$$\phi(z,\theta,m) = \sum_{k=0}^{\infty} \frac{\left(\lambda\pi D^2\Omega\right)^k}{k!} e^{-\lambda\pi D^2\Omega} \times \times \left(\int_{D_0}^{D} \frac{1}{N} \sum_{l=1}^{N} e^{-r_1{}^{-\beta} z \frac{1}{l^2\pi^2} C_1(\theta)} \frac{2r_1 dr_1}{D^2 - D_0^2} \right)^k \tag{20}$$

with $\Omega = N \cdot \Delta f_c = N/T_s$ investigated frequency domain. Upon exploiting that $\sum_{k=0}^{\infty} \frac{A^k}{k!} e^{-A} B^k = e^{A(B-1)}$ we get

$$\phi(z,\theta,m) = = \exp\left(\frac{\lambda\pi D^2}{T_s} \left(\sum_{l=1}^{N} \int_{D_0}^{D} r_1 e^{-\frac{r_1{}^{-\beta}}{l^2} K_1(\theta,m)} \frac{2dr_1}{D^2 - D_0^2} - N \right) \right) \tag{21}$$

where $K_1(\theta,m) = z\frac{1}{\pi^2} C_1(\theta,m)$. Let us also exploit that

$$\int_{D_0}^{D} r_1 e^{-\frac{r_1{}^{-\beta}}{l^2} K_1(\theta,m)} dr_1 = = \left[\frac{r_1^2}{\beta} \left(\frac{r_1{}^{-\beta} K_1(\theta,m)}{l^2} \right)^{\frac{2}{\beta}} \Gamma\left(-\frac{2}{\beta}, \frac{r_1{}^{-\beta} K_1(\theta,m)}{l^2} \right) \right]_{D_0}^{D} \tag{22}$$

where $\Gamma(\rho,a) = \int_a^{\infty} e^{-t} t^{\rho-1} dt$ is defined as the incomplete Gamma function. After substituting (22) into (20) we get the BER curves of Fig. 5 which illustrated the BER curves for 4-, 8- and 16-PSK with Rice-fading also in this case with $\kappa = 10$. The $\lambda = 10^{-4}$ and 10^{-5} values have been assumed for the spatial and frequency-domain density of the interferers, resulting in a higher interference and lower BER for larger λ settings. As it was observed at the previous cases, higher interference effects degrade the BER results for increasing M values.

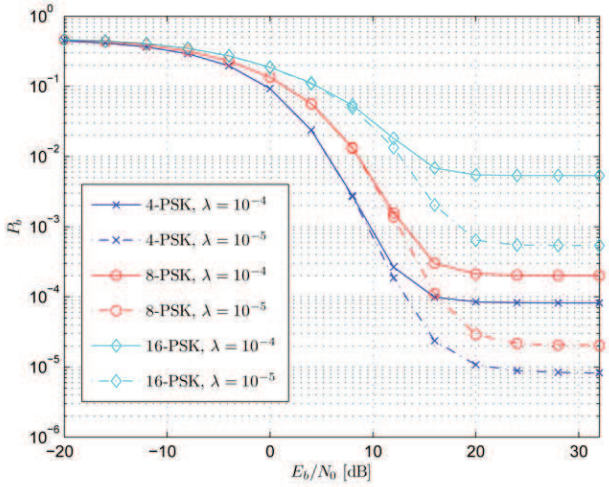

Fig. 5: BER curves for 4-, 8- and 16-PSK in case of Rice-fading with $\kappa = 10$ and different density values of interferers $\lambda = 10^{-4}$ and 10^{-5}

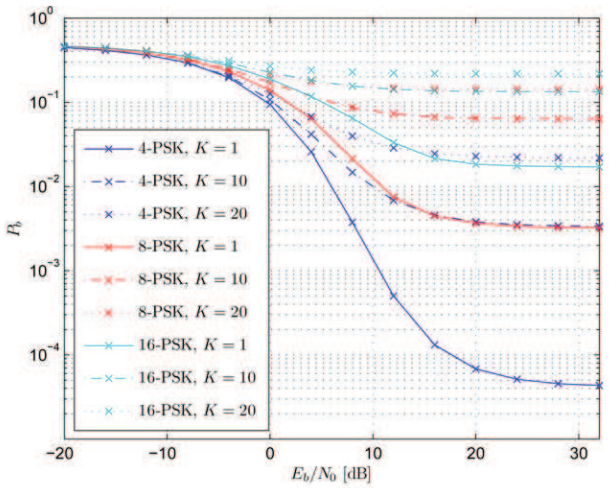

Fig. 6: BER curves for 4-, 8- and 16-PSK in case of Rice-fading with $\kappa = 10$ and different $K = 1$, 10 and 20 number of interferers

C. Interferers with raised cosine spectral shapes

For this scenario we assume $v_{rc}(\Delta f_k)$ defined for raised cosine (RC) in (10). We will perform an analysis by assuming Δf_k to be RVs with uniform distribution within the interval $[0, \frac{2}{T_s})$. Accordingly, we will calculate the expected value within $\phi(z,\theta,m|\Delta f_k, s_k)$ with assuming $s_k = s_1$, $\Delta f_k = \Delta f_1$ $\forall k$ and $s_1/s_0 = 10^{-\frac{\alpha_{dB}}{10}}$ as

$$\phi(z,\theta,m|s_1) = \left(2\int_0^{\frac{2}{T_s}} e^{-z\frac{s_1}{s_0 T_s} v_{rc}(\Delta f_1) C_1(\theta,m)} \frac{d\Delta f_1}{\Omega} \right)^K = = \left(\frac{2}{\Omega} \int_0^{\frac{2}{T_s}} e^{-z\frac{s_1}{s_0 T_s} v_{rc}(\Delta f_1) C_1(\theta,m)} d\Delta f_1 \right)^K = = \left(T_s \int_0^{\frac{2}{T_s}} e^{-z\frac{10^{-\frac{\alpha_{dB}}{10}}}{T_s} v_{rc}(\Delta f_1) C_1(\theta,m)} d\Delta f_1 \right)^K, \tag{23}$$

in which $\Omega = \frac{2}{T_s}$. The BER curves can be observed in Fig. 6 for 4-, 8- and 16-PSK with different number of the interferers K. Since the investigated $[0, \frac{2}{T_s})$ interval for the RC spectral shapes means a rather scant domain around the center frequency f_0 of the useful signal and in our model all f_k are located within that, the resultant interference effects leading to a BER degradation would be higher than in the case of the sinc spectral shapes (Fig. 19).

VII. CONCLUSIONS

An exact closed form calculation method was provided for the BER calculation of M-PSK transmission, which considers the interference and fading effects alike. A high-flexibility interference model has been embedded into an analytic closed form BER calculation framework, which has been originally developed for AWGN channel and without the consideration of any interference effects. The demonstrated calculation method

evaluates the effects of several input parameters, i.e. the received power of the useful signal, the received power values and center frequencies of the interferers *as random variables*, and calculates the BER as an average with respect of many RVs. This way of calculation gives a solution for such complex fading and interference scenarios which could be investigated only by simulations up to this point.

APPENDIX A
VALIDATION OF THE BER CURVES

Let introduce the ρ value of the Signal-to-Interference-Ratio (SIR) expressed as

$$\rho = \frac{s_0}{\sum\limits_{k=1}^{K} s_k} = \frac{s_0}{K s_1} \tag{24}$$

if all of the the interferers have equal $s_k = s_1 \ \forall k$. After that $s_0 = \rho \cdot K s_1$. Our goal is now to validate the BER results with the help of paper [8] for a *deterministic* case (for s_1 and s_0), since the results of it have already been proofed by simulations and also embedded into the MATLAB®Communications Toolbox.

Let us execute an SNR conversion, which will be resulted as an interference accession according to equation (11). After that, we will get certainly lower effective SNR values, which should be substituted as an input of the expression in [8].

Let denote the 'original' (interference-free) SNR with $\gamma = \frac{E_b}{N_0}$, from which and (24)

$$\gamma = \frac{E_b}{N_0} = \frac{s_0 T_b}{N_0} = \frac{\rho \cdot K s_1}{N_0}$$

According to expressions (15) and (16), we will consider the SNR degradation caused by the interference in an effective γ' SNR expression as

$$\begin{aligned} \gamma' &= \frac{E_b}{N_0 + \sum\limits_{k=1}^{K} s_1 v(\Delta f_k)} = \frac{\gamma}{1 + \frac{s_1}{N_0} \sum\limits_{k=1}^{K} v(\Delta f_k)} = \\ &= \frac{\gamma}{1 + \gamma \frac{1}{\rho K T_b} \sum\limits_{k=1}^{K} v(\Delta f_k)}. \end{aligned} \tag{25}$$

If we substitute γ' into the expressions of [8], we will get and validate the BER curves. Note that for plotting the horizontal axis should contain the 'original' γ values. However, our work is not only a reproduction (or unnecessary complication) of [8], since we have provided a tool for the stochastic handling of s_0, s_k and f_k parameters.

REFERENCES

[1] A. Mraz and L. Pap, "General performance analysis of M-PSK and M-QAM wireless communications applied to OFDMA interference," in *Wireless Telecommunications Symposium (WTS), 2010*, 2010, pp. 1 –7.

[2] K. Hamdi and L. Pap, "A unified framework for interference analysis of noncoherent MFSK wireless communications," *Communications, IEEE Transactions on*, vol. 58, no. 8, pp. 2333 –2344, 2010.

[3] A. Mraz and L. Pap, "General interference analysis of M-QAM transmission applied to LTE performance evaluation," in *EUROCON - International Conference on Computer as a Tool (EUROCON), 2011 IEEE*, april 2011, pp. 1 –4.

[4] D. Y. Kyongkuk Cho, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Transactions on Communications*, vol. 50, no. 7, pp. 1074–1080, July 2002.

[5] D. Y. Jaeyoon Lee and S. K. Park, "Further result on the symbol error probability of MPSK with I/Q phase unbalance," *IEICE Transactions on Communications*, vol. E89-B, no. 5, pp. 1675–1677, May 2006.

[6] D. Woojin Jeong, Jaeyoon Lee, "New BER expressions of MPSK," *submitted to IEEE Transactions on Communications*, Mar 2010.

[7] P. Lee, "Computation of the bit error rate of coherent M-ary PSK with gray code bit mapping," *Communications, IEEE Transactions on*, vol. 34, no. 5, pp. 488 – 491, may 1986.

[8] J. Lassing, E. Strom, E. Agrell, and T. Ottosson, "Computation of the exact bit-error rate of coherent M-ary PSK with gray code bit mapping," *Communications, IEEE Transactions on*, vol. 51, no. 11, pp. 1758 – 1760, nov. 2003.

[9] J. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill Inc., 1995.

László Pap (M93-SM97) graduated from the Technical University of Budapest, Faculty of Electrical Engineering, Branch of Telecommunications. He became Dr. Univ. and Ph.D. in 1980, and Doctor of Sciences in 1992. In 2001 and 2007 he has been elected as a Corespondent and Full Member of the Hungarian Academy of Sciences. His main fields of the research are the electronic systems, nonlinear circuits, synchronization systems, modulation and coding, spread spectrum systems, ATM, CDMA, multiuser detection and mobile communication systems. His main education activity has covered the fields of electrical amplifiers and circuits, electronics, modern modulation and coding systems, communication theory, introduction to mobile communication, transmission and processing of information, nonlinear circuits and digital technique. Professor Pap had been Head of the Dept. of Telecommunications, Budapest University of Technology and Economics and the Vice-Rector of the University.

Albert Mráz (M2005) had received the MSc degree in electrical engineering from the Budapest University of Technology and Economics (BUTE) in 2005. He is currently working toward the Ph.D. degree at BUTE. His research interests include the modeling and resource management of broadband wireless networks (3G and 4G).