

# Determination of Low Pass Filter Coefficients for Receiver with Zero Intermediate Frequency by Differential Evolution Algorithm

Martin Vestenický, Peter Vestenický, Vladimír Hottmar

**Abstract** — In the paper the method of a low pass filter design for a receiver with zero intermediate frequency is presented. Definition of design criteria is performed on the system level instead of specific filter features determination. The design process uses differential evolution algorithm. Main part of the paper describes a creation of cost function and the method of its application in cooperation with the used algorithm. Conclusion of the paper presents a found solution and its comparison with commonly used filters.

**Index Terms** — differential evolution, low pass filter, transfer function, stochastic algorithm

## I. INTRODUCTION

THE continuously increased demands on the electronic circuits being used in telecommunications impose raising requirements on their optimal design taking technological, economical, construction and other limitations into account. Therefore research must be focused on new methods for optimized design of electronic circuits [2]. One of new approaches to this challenge are the stochastic algorithms. These algorithms are being used to numerical solving of complex optimization problems. The main advantage of stochastic algorithms is the easy applicability in many areas of research and design. Next major feature is the ability of effective solving of multi-criterion optimization problems with a large number of independent variables. One of application fields for using the stochastic algorithms is the optimized design of functional blocks for communication systems.

This paper deals with determination of coefficients for transfer function of a low pass filter which is applied in a zero intermediate frequency receiver whose block diagram is shown in Fig. 1. Such receiver structure has been published in [4]. In the design process the well known algorithm of differential evolution (DE) described in detail in [1] has been used.

Martin Vestenický, Faculty of Electrical Engineering, Department of Telecommunications and Multimedia, University of Žilina. Univerzitná 8215/1, 010 26 Žilina, Slovakia (e-mail: martin.vestenicky@fel.uniza.sk).

Peter Vestenický, Faculty of Electrical Engineering, Department of Control and Information Systems, University of Žilina. Univerzitná 8215/1, 010 26 Žilina, Slovakia (e-mail: peter.vestenicky@fel.uniza.sk).

Vladimír Hottmar, Faculty of Electrical Engineering, Department of Telecommunications and Multimedia, University of Žilina. Univerzitná 8215/1, 010 26 Žilina, Slovakia (e-mail: vladimir.hottmar@fel.uniza.sk).

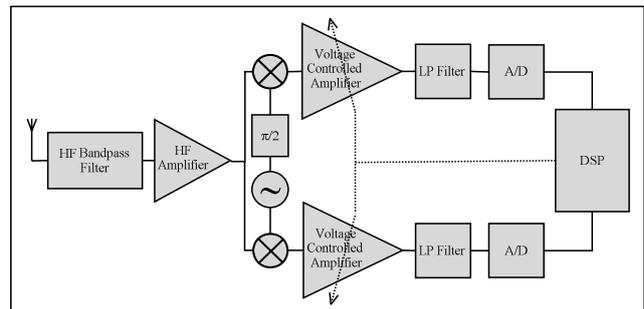


Fig. 1. Block diagram of receiver with zero intermediate frequency

## II. APPLICATION OF DE FOR DETERMINATION OF FILTER COEFFICIENTS

The application objective of DE algorithm is to determine the coefficients of a low pass filter transfer function which for the given type of modulation and for the given signal-to-noise ratio minimizes the symbol error ratio of data transfer between transmitter and receiver taking some limitation into account: fixed order, causality and stability of filter. This type of task is in [4] referred to as highly topical.

For this application the sixth order switched capacitor or switched current filter is assumed. The transfer function of this filter is given by formula (1). For using DE algorithm the association of particular variables (filter coefficients) to chromosomes of individual have to be defined (Fig. 2). Therefore, the individual is represented by fourteen element vector where every element represents one coefficient of a searched transfer function in the form of real number without other limitations.

$$H(z) = \frac{b_6 \cdot z^6 + b_5 \cdot z^5 + b_4 \cdot z^4 + b_3 \cdot z^3 + b_2 \cdot z^2 + b_1 \cdot z + b_0}{a_6 \cdot z^6 + a_5 \cdot z^5 + a_4 \cdot z^4 + a_3 \cdot z^3 + a_2 \cdot z^2 + a_1 \cdot z + a_0} \quad (1)$$

The only modification of DE algorithm compared to the version described in [1] is the method of a new individual creation which is given by formula (2). This modification which is in detail described in [6] enabled a particular increase of convergence speed against the original implementation of DE algorithm.

## Determination of Low Pass Filter Coefficients for Receiver with Zero Intermediate Frequency by Differential Evolution Algorithm

The formula (2) describes the creation of  $j$ -th chromosome of a new individual as linear combination of  $j$ -th chromosomes of three randomly selected individuals from current population and  $j$ -th chromosome of the best created individual till. The algorithm input constants are summarized in Table I and their meaning is explained in detail in [1], [5], [6] and [7]. The values of constants have been selected taking conclusions from [5] into account. The criterion for algorithm termination is the maximum number of generations  $G_{max}$ .

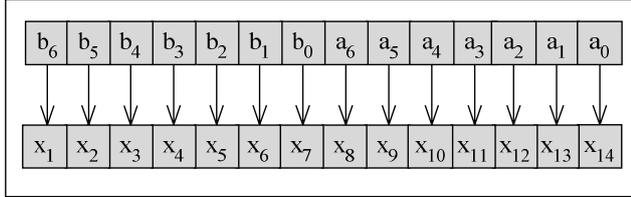


Fig. 2. Association of variables to chromosomes of individual

$$temp_j = \begin{cases} POP_{n,j,G} + F_1 \cdot (POP_{n,j,G} - POP_{r,j,G}) + \\ \quad + F_2 \cdot (best_j - POP_{n,j,G}) & \text{if } rand_j[0,1] < CR \vee j = j_{rand} \quad j = 1, \dots, D \quad (2) \\ POP_{n,j,G} & \text{else} \end{cases}$$

### III. DEFINITION OF COST FUNCTION

The most complex problem in solving such a type of design task is to define the cost function. In this case the cost function has been built by a non-standard method, namely by using the simulation model of a whole transfer chain which is shown in Fig. 3. The model serves for searching close-optimal coefficients of a transfer function given by the general formula (1). This model consists of three basic parts:

- The transmitter of QAM signal which generates random QAM signal with a defined number of states,
- Data transmission channel which simulates signal transmission in real environment, in this case the modified AWGN (Additive White Gaussian Noise) channel was used,
- The receiver whose part is the designed filter. Other functional blocks are parts of the receiver, too.

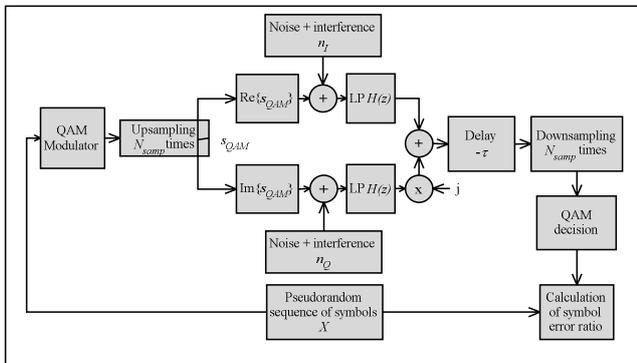


Fig. 3. System model of transfer chain

The model uses signal processing in base band for simplicity. It does not cause significant deviation of the results compared with reality [3]. The model is universal with respect to the number of QAM states  $M$ . It works with pseudorandom sequence of symbols  $X$  which can be assumed as a sequence of pseudorandom integer numbers from interval  $\langle 0, M-1 \rangle$  where  $M$  is the number of QAM modulation states. This sequence is used as an input parameter for the mathematical model of QAM modulator which associates the corresponding points in signal space to particular members of the sequence according to a predefined signal point constellation. By this procedure the QAM modulated signal is being created in base band which is mathematically represented as a complex number sequence. The length of this sequence is the same as the length of the sequence  $X$ . It is necessary for every symbol from the sequence  $X$  to assign  $N_{samp}$  of samples. In that way the sequence of signal samples  $s_{QAM}$  is created. The signals assigned to I (In phase) and Q (Quadrature) planes are obtained by splitting of every  $s_{QAM}$  sequence member on real and imaginary components.

The transfer via the radio channel is mathematically modeled by adding of signals representing noise and interference which are given as the sequence of samples  $n_I$  and  $n_Q$  for I and Q planes respectively. In that way the testing signals have been created and the ones will be used for testing of solution represented by actual transfer function  $H(z)$ . After processing of both signals from I and Q planes by low pass filters and after mathematical elimination of filter delay  $\tau$  the signal samples are added. Before adding the samples from the Q plane they are multiplied by imaginary unit  $j$ . The calculation of delay  $\tau$  is given by formula (3), where  $N$  is the number of samples of the received signal  $T_{sig}$  before filtration,  $Pad$  is the number of zero samples which must be added to the original sequences for correct working of the simulation of filtering process, and  $T_{sigfilt}$  is the received signal after filtration.

$$\tau = N + Pad - \max \left( \text{corr} \left( \vec{T}_{sig}, \vec{T}_{sigfilt} \right) \right) \quad (3)$$

The sequence of complex number has been created by the described procedure. This sequence is in the next step downsampled  $N_{samp}$  times. A new sequence of complex numbers has its length identical with the sequence  $X$  and represents the sequence of points in signal space at receiving side. It is an input argument for the mathematical model of QAM decision. The QAM decision assigns an integer number from interval  $\langle 0, M-1 \rangle$  to every member of the input sequence and creates a sequence of received symbols. By comparing both transmitted and received sequences the symbol error ratio  $SER$  is calculated by formula (4). The symbol error ratio is the value of a cost function for actual solution represented by coefficients of the transfer function  $H(z)$ .

$$SER = \frac{N_{err}}{N_{symp}} \quad (4)$$

In the course of DE algorithm the cost function has been calculated for every new originated individual by the previously described process. At every cost cycle approximately  $N_{symp}=10^4$  of symbols were transferred.  $N_{err}$  is the number of incorrectly received symbols. This value is a compromise between calculation time and accuracy of cost function.

TABLE I  
VALUES OF CONTROL CONSTANTS

Parameters of model			Control constants of DE algorithm		
Transfer rate	$DR$	1 Mbps	Number of unknown quantities	$D$	14
Number of symbols	$N_{symp}$	10000	Maximum number of generations	$G_{max}$	100
Number of samples per symbol	$N_{samp}$	8	Number of population individuals	$NP$	280
Number of modulation states	$M$	16	Crossover ratio	$CR$	0.9
Signal - to - noise ratio	$SNR$	18 dB	Weighting coefficient of differential mutation 1	$F1$	0.5
Output signal power	$P_{out}$	1 W	Weighting coefficient of differential mutation 2	$F2$	0.5
Frequency boundary of interference	$f$	275 kHz			

IV. RESULTS AND DISCUSSION

Applying the DE algorithm, the transfer function (5) which does not correspond to any standard approximation was found. Moreover, no requirements for the filter must be predefined except for its order.

$$H(z) = \frac{1.0722 \cdot z^6 + 1.7945 \cdot z^5 + 2.5056 \cdot z^4 + 2.6168 \cdot z^3 + 2.7587 \cdot z^2 + 1.7411 \cdot z + 1.5783}{43.170 \cdot z^6 - 33.247 \cdot z^5 + 1.2873 \cdot z^4 - 4.9585 \cdot z^3 + 4.8114 \cdot z^2 + 1.8533 \cdot z + 0.66119} \quad (5)$$

The requirements for filter stability and causality are tested during the optimizing process. These requirements are necessary for its correct operation and ability to be realized by circuits based on switched capacitor or switched currents technology. The check analysis indicates that the found transfer function (5) is stable and causal. Frequency response of the found filter is shown in Fig. 4 together with frequency response of the matched filter which is calculated for rectangular pulses and modulation rate  $DR/\log_2 M=250$  k Bd (see Table I).

In Figs. 5a and 5b the constellations of signal points of the testing signal and signal after filtering by a filter with transfer function (5) are presented. These constellations were created for  $10^4$  symbols and for signal - to - noise ratio 18 dB.

By using of the found filter the bit error ratio is significantly decreased (about one or two orders) in comparison with filters

with standard approximations. This comparison is shown in Fig. 6a and Fig. 6b. In this case the simulation was performed for a communication channel with power spectral density of noise which is shown in Fig. 7, i. e. it is not a standard AWGN channel. Theoretical estimation of the bit error ratio for assumed type of modulation was performed according to the formula (6), where  $E_s$  is the energy of one symbol and  $N_0$  is the power spectral density of added white noise. The bit error ratio for all simulations was calculated from the symbol error ratio by the formula (7), where  $M$  is the number of modulation states. Selection of filters for comparison was inspired by experiments presented in [3], [4]. Parameters of compared filters are summarized in Table II.

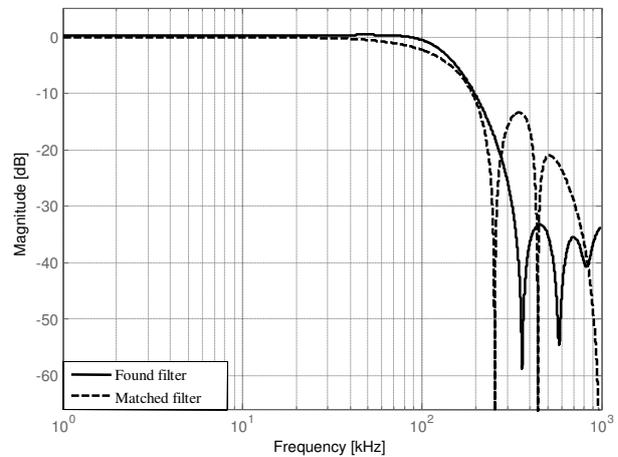


Fig. 4. Frequency response of found filter and matched filters

$$BER_{Theor} = \frac{2 \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot \operatorname{erfc} \left( \sqrt{\frac{3}{2 \cdot (M-1)} \cdot \frac{E_s}{N_0}} \right) - \left(1 - \frac{1}{\sqrt{M}}\right)^2 \cdot \operatorname{erfc}^2 \left( \sqrt{\frac{3}{2 \cdot (M-1)} \cdot \frac{E_s}{N_0}} \right)}{\log_2 M} \quad (6)$$

$$BER = \frac{SER}{\log_2 M} \quad (7)$$

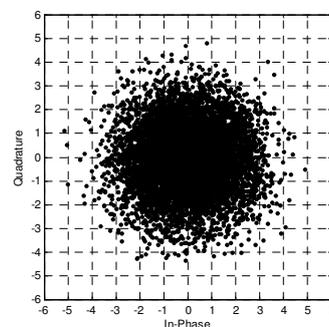


Fig. 5a. Constellation diagram before filtration

Determination of Low Pass Filter Coefficients for Receiver with Zero Intermediate Frequency by Differential Evolution Algorithm

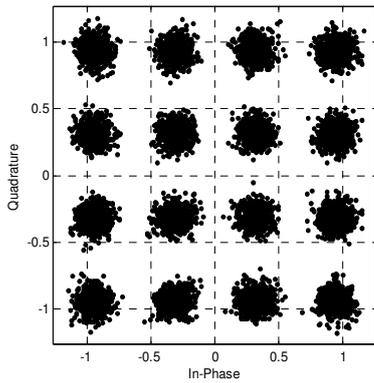


Fig. 5b. Constellation diagram after filtration

TABLE II  
PARAMETERS OF COMPARED FILTERS

Type of filter	Filter order	Cut-off frequency [kHz]	Passband ripple [dB]	Stopband ripple [dB]	Roll-off factor
Inverse Chebyshev	6	250	-	40	-
Cauer	6	250	0.1	44	-
Raised cosine	6	-	-	-	0.5
Matched filter	6	250	-	-	-
Theoretical guess	-	250	-	-	0
Found filter	6	-	-	-	-

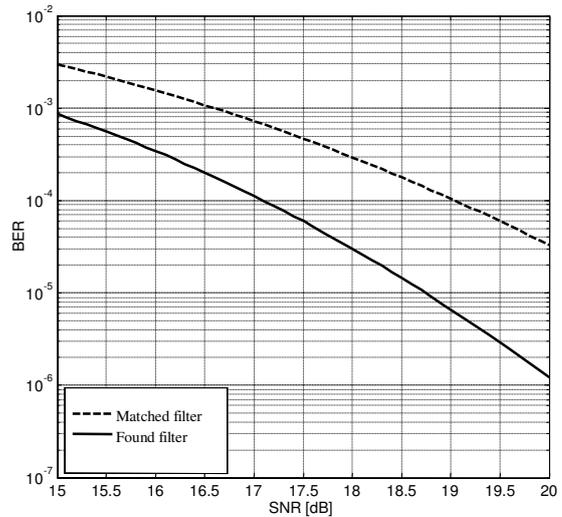


Fig. 6b. Comparison of BER for matched filter, found filter and non-standard noise channel

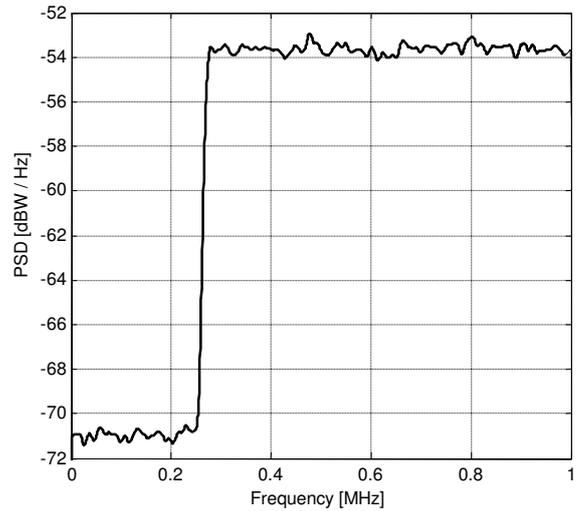


Fig. 7. Power spectral density for non-standard noise channel

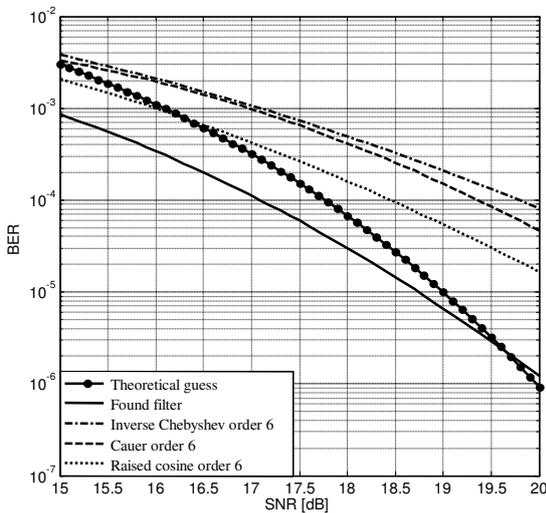


Fig. 6a. Comparison of BER for various filters and non-standard noise channel

For comparison, simulation of BER was performed for standard AWGN channel, too. The results are shown in Fig. 8 for the found filter and matched filter.

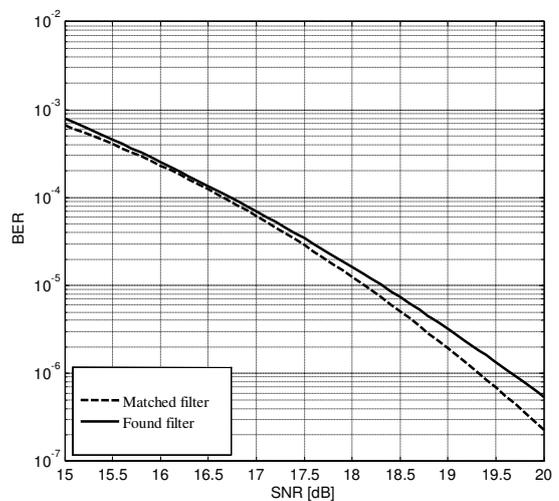


Fig. 8. Comparison of BER for standard AWGN channel

V. CONCLUSION

Application of stochastic algorithms enabled to put into the filter design various and often contradictory requirements which are hardly implementable simultaneously by standard design procedures. Based on the attained results it can be assumed that stochastic algorithms are applicable to the other functional blocks of the system, too. Essentially the whole system can be designed based only on predefined requirements on the functionality of whole system without definition of requirements for its particular functional blocks. Slight disadvantage of stochastic algorithms is a considerable time consumption of the design process. This disadvantage can be eliminated by parallel processing of stochastic algorithm described in [7].



**Vladimír Hottmar** graduated from the University of Žilina in 1975 with the Master's degree in Transport and Communications. At the beginning of his career he was employed at the Research Institute of Computers Techniques in Žilina, where he worked as a researcher at the Department of Microcomputers and Development Systems for 22 years, later being responsible for running the Research Department. In 1999 he accomplished his Ph.D. studies at the University of Žilina. Currently he is

employed at the University of Žilina at the Department of Telecommunications and Multimedia in the position of associate professor and is also involved in some research work. He is a VEGA project leader.

REFERENCES

[1] Corne, D., Dorigo, M., Glover, F., "New Ideas in Optimization," McGraw-Hill, London 1999, ISBN 001-709506-5  
 [2] Craninckx J., Donnay S., "4G Terminals: How Are We Going to Design Them?" Proceedings of the 40th Design Automation Conference 2003, pp. 79 - 84, June 2003, ISBN 1-58113-688-9  
 [3] Emira A. A., Valdes-García A., Xia B., Mohieldin A. N., Valero-Lopez A. Y., Moon S. T., Xin C., Sánchez-Sinencio, E., "Chameleon: A Dual Mode 802.11b/Bluetooth Receiver System Design." *IEEE Transaction on Circuits and Systems - I: Regular Papers*, Vol. 53, No. 5, pp. 977-984, May 2006, ISSN 1057-7122  
 [4] Korotkov A. S., Morozov D. V., Tutyshkin A. A., Hauer H., "Channel Filters for Microelectronic Receivers of Wireless Systems." IEEE 7th CAS Symposium on Emerging Technologies: Circuits and Systems for 4G Mobile Wireless Communications, pp. 24 - 31, St. Petersburg State Polytechnic University, Russia 2005, ISBN 5-7422-0895-2  
 [5] Price, K., Storn, R., M., Lampinen, J., A., "Differential Evolution: A Practical Approach to Global Optimization." Springer, 2005. ISBN 978-3-540-20950-8.  
 [6] Vestenický, M., "The Use of Evolutionary Algorithms in Selective Systems Synthesis for Communication Systems." Dissertation thesis, University of Žilina, Faculty of Electrical Engineering, 2007 (in Slovak)  
 [7] Vestenický M., Vestenický P., "The Use of MDCE Environment for Parallel Processing of Stochastic Algorithms." *Annals of Faculty Engineering Hunedoara – International Journal of Engineering*, vol. 8, nr. 2/2010, pp. 55 - 60. ISSN 1584-2665



**Martin Vestenický** was born in Martin, Slovakia in 1980. He received his Ing. (M.Sc.) degree in 2003 and his Ph.D. degree in 2008 both at the Department of Telecommunications and Multimedia, University of Žilina. He joined the Department of Telecommunications and Multimedia, University of Žilina in 2006. His recent research interests are stochastic algorithms based optimization, intelligent transportation systems, and sensors networks.



**Peter Vestenický** was born in 1967. He received Ing. (M.Sc.) degree in 1990 at the Department of Telecommunications and Multimedia, University of Žilina and the Dr. (Ph.D.) degree in 1994 at the same department. His Ph.D. thesis was focused on the Asynchronous Transfer Mode (ATM) and mathematical modelling of policing mechanisms for ATM networks. Since 1994 he has been working at the Department of Control and Information Systems, University of Žilina.

His main research areas are radiofrequency identification, intelligent transportation systems, industrial and computer communication networks.