Energy balancing by combinatorial optimization for wireless sensor networks

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The paper is concerned with developing new energy balancing protocols for wireless sensor networks (WSN) to maximize the life-span of the system by using rare event tools. When developing these new protocols, the statistical traffic characteristics of the sensed quantities are taken into account and some novel packet forwarding mechanisms from the nodes to the base station (BS) are proposed, which minimize the energy consumption of WSN. The tail distribution of the energy consumption is estimated by the tools of large deviation theory and the concept of generalized statistical bandwidth has been introduced to evaluate the energy need of the network. Furthermore, the clusterhead (CH) selection of "LEACH-like" protocols have been optimized by using spanning tree design and improved Li-Silvester bounds. The new results have been tested by extensive simulations which demonstrated that the lifespan of WSN can significantly be increased by the new protocols.

1. Introduction

Due to the recent advances in electronics and wireless communication, the development of low-cost, low-power, multifunctional sensors have received increasing attention [1]. These sensors are compact in size and besides sensing they also have some limited signal processing and communication capabilities. However, these limitations in size and energy make WSNs different from other wireless and ad-hoc networks [2]. As a result, new protocols must be developed with special focus on energy balancing in order to increase the lifetime of the network, which is crucial in applications where recharging of the nodes is out of reach (e.g. military field observations, living habitat monitoring etc., for more details see [4]).

The paper addresses energy balancing in WSN and develops novel packet forwarding mechanisms to increase the lifetime of the system. First a random class of protocols will be investigated, where the sensor nodes randomly select other nodes for packet forwarding, subject to a probability distribution. For example, node i can choose to forward to the neighbouring node closer to the base station (labeled as i-1) with probability $1-a_i$, or send the packet directly to the BS with probability a_i. The optimal p.d.f. a_i, i=1,...,N is found which maximizes the tail of life-time distribution, based on large deviation theory by extending the concept of statistical bandwidth.

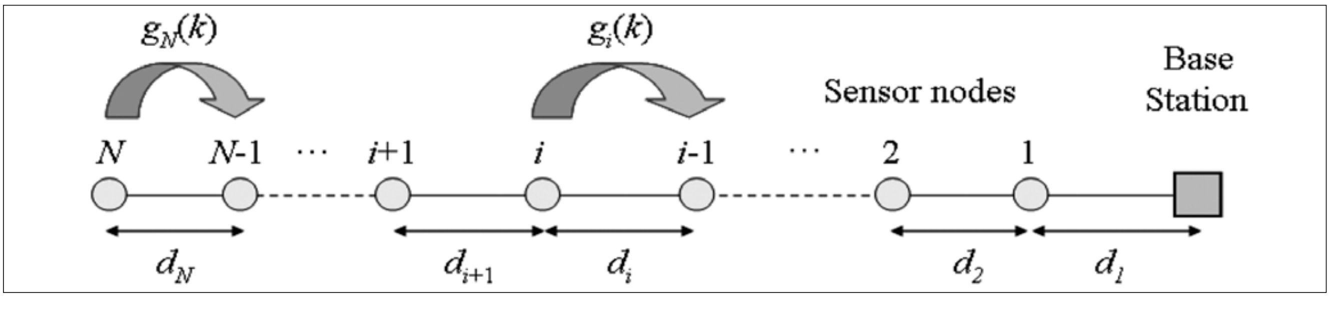
Then a LEACH-type protocol is analyzed. In this case, the active nodes select a cluster-head to which all the generated packet are sent and then the CH re-transmits the received packets to the BS. However, as opposed to the random CH selection of the traditional LEACH protocol (detailed in [6]), we select the CH by using an optimal spanning tree model. This spanning tree statistically optimizes the minimum remaining energy over all possible random traffic state vectors. Since the design of such a spanning tree is of exponential complexity, we develop a modification of the Li-Silvester bounds (which is known in statistical reliability analysis for reliability measure estimation) to optimize the protocol.

The new protocols can ensure longer WSN lifespan than the traditional packet forwarding mechanisms which is also demonstrated by extensive simulations.

2. The model

After the routing protocol (e.g. LEACH [6-8] or PEDAP [7]) has found the path to the base station, the subsequent nodes participating in the packet transfer can be regarded as a one dimensional chain labeled by i= 1,...,N and depicted by *Figure 1*.

Figure 1.
One dimensional chain topology of WSN packet forwarding



The system is characterized as follows:

- the topology is uniquely defined by a distance vector $\mathbf{d}=(d_1,...,d_N)$, where $d_i, i=1,...,N$ denotes the distance between node i and i-1, respectively;
- the energy needed to transmit packet over distance d is given as $g=\frac{d^\alpha \Theta \sigma_Z^2}{-\ln p_r}+g_{Elec}$ dictated by the Rayleigh model, where d is the distance, α depends on the propagation type, p_r is the reliability of correct reception, Θ is the modulation coefficient, σ_Z^2 is the noise energy, while g_{Elec} represents the consumption of the electronics during transmitting and receiving;
- the initial battery power on each node is the same and denoted by C;
- we assume that each sensor generates packets subject to an On/Off model, i.e. packet generation occurs with probability $P(y_i=1)=p_i$, whereas the node does not generate packet with probability $P(y_i=0)=1-p_i$;
- the traffic state of the network is represented by an N dimensional binary vector $\mathbf{y}\in\{0,1\}^N$ and the corresponding probability of a traffic state is given as $p(\mathbf{y})=\prod_{i=1}^N p_i^{y_i}(1-p_i)^{1-y_i}$ assuming independence among the sensed quantities;
- the nodes operate in a time synchronous manner where the discrete time (clock signal) is denoted by $k=0,1,2,...$

As a result, a WSN is fully characterized by vectors **g**, **p** and **c**, respectively.

When analyzing the lifespan of the network, the following packet forwarding mechanisms are taken into account:

1. *Chain protocol:*
   Each node transmits packet to its neighbour laying closer to the BS. In this way, each node consumes minimal energy being engaged with short range energy transmission. However, each packet is traversing toward the BS, thus a packet consumes energy on each node along its path to the BS.
2. *Random shortcut protocol:*
   Node i can choose to forward the packet to its neighbouring node closer to the base station (labeled as i-1) with probability $1-a_i$, or directly send the packet to the BS with probability a_i.
3. *Single-hop protocol:*
   Each node sends its packet directly to the BS.
4. *CH protocol:*
   Each active node forwards its packet to a selected cluster-head and CH re-transmits them to the BS.

The paper is concerned with evaluating the lifetime of these protocols. Furthermore, our aim is to optimize probability vector $\mathbf{a}=(a_1,...a_N)$ and the CH selection in order to minimize energy consumption and thus maximizing the lifespan for WSNs operating with the random shortcut protocol.

3. Lifespan estimation by large deviation theory

Let assume that the chain protocol is in effect. The energy consumed by sending a packet generated on node i to the BS is given as

$$G_i := \sum_{j=1}^{i} g_j , \tag{1}$$

and the average energy consumption up to time instant K is given as

$$\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i(k)G_i . \tag{2}$$

The lifespan of node denoted by $\tilde{K}$ is defined as

$$\tilde{K}: P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i(k)G_i < C\right)=e^{-\alpha} , \tag{3}$$

where $e^{-\alpha}$ is close to one and
α is a reliability parameter.

By using the complementary probability

$$P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i(k)G_i > C\right)=1-e^{-\alpha} , \tag{4}$$

life time evaluation is cast as a tail estimation problem, where bounds like the Chernoff inequality can be used as (5):

$$P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i(k)G_i > C\right)\le \exp\left(\sum_{i=1}^{N}\mu_i(\hat{s},G_i)-\frac{\hat{s}NC}{K}\right)$$

Here $\mu_i(s,G_i):=\log\left(\mathrm{E}\left[e^{sy_iG_i}\right]\right)=\log\left(1-p_i+p_ie^{sG_i}\right)$

and $\hat{s}:\min_s K\sum_{i=1}^{N}\mu_i(s,G_i)-\frac{sNC}{K}$.

By using the estimation above, one obtains

$$e^{\sum_{i=1}^{N}\mu_i(\hat{s},G_i)-\frac{\hat{s}NC}{K}}=1-e^{-\alpha} \tag{6}$$

and the lifespan of the simple chain protocol can finally be estimated by the following formula:

$$\tilde{K}=\frac{\hat{s}NC}{\sum_{i=1}^{N}\mu_i(\hat{s},G_i)+\log\left(1-e^{-\alpha}\right)} . \tag{7}$$

If the random shortcut protocol is in effect, then the packet generated by node i will travel in the chain down to the fits shortcut to BS. Let the node in which the shortcut takes place is denoted by λ_i. The distribution of λ_i is given as

$$P(\lambda_i=l_i)=a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right) . \tag{8}$$

In this case the packet consumes $V_i:=\sum_{j=i-l_i+1}^{i} g_j+\gamma_{i-l_i}$ energy, where γ_{i-l_i} is the shortcut energy from node $i-l_i$ (i.e. the energy required to transmit the packet from node $i-l_i$ directly the BS). As a result, the average energy consumption is given as

$$\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-\lambda_i+1}^{i} g_j+\gamma_{i-\lambda_i}\right) . \tag{9}$$

Thus the lifespan is defined as follows:

$$\tilde{K}: P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-\lambda_i+1}^{i} g_j+\gamma_{i-\lambda_i}\right)>C\right)=1-e^{-\alpha} \quad (10)$$

The probability in equation (10) can be rewritten as

$$P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-\lambda_i+1}^{i} g_j+\gamma_{i-\lambda_i}\right)>C\right)=$$

$$=\sum_{l_1}\cdots\sum_{l_N} P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-\lambda_i+1}^{i} g_j+\gamma_{i-\lambda_i}\right)>C \,\middle|\, \lambda_1=l_1,\ldots,\lambda_N=l_N\right)P\left(\lambda_1=l_1,\ldots,\lambda_N=l_N\right)=$$

$$=\sum_{l_1}\cdots\sum_{l_N} P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-l_i+1}^{i} g_j+\gamma_{i-l_i}\right)>C\right)\prod_{i=1}^{N}P\left(\lambda_i=l_i\right)\le$$

$$=\sum_{l_1}\cdots\sum_{l_N} e^{\sum_{i=1}^{N}\mu_i(s,V_i)-\frac{sNC}{K}}\prod_{i=1}^{N}\left(a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right)\right)=e^{-\frac{sNC}{K}}\sum_{l_1}\cdots\sum_{l_N}\prod_{i=1}^{N}e^{\mu_i(s,V_i)}\left(a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right)\right)$$

$$=\sum_{l_1}\cdots\sum_{l_N} e^{\sum_{i=1}^{N}\mu_i(s,V_i)-\frac{sNC}{K}}\prod_{i=1}^{N}\left(a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right)\right)=e^{-\frac{sNC}{K}}\prod_{i=1}^{N}\sum_{l_i}\left(e^{\mu_i(s,V_i)}\left(a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right)\right)\right),$$

where $\mu_i\left(s,V_i\right):=\log\left(\mathrm{E}\left[e^{sy_iV_i}\right]\right)=\log\left(1-p_i+p_ie^{sV_i}\right)$.

Introducing the extended logarithmic moment generation function as

$$\beta_i\left(s,V_i\right):=\log\left(\sum_{l_i}\left(e^{\mu_i(s,V_i)}a_{i-l_i}\prod_{j=i-l_i+1}^{i}\left(1-a_j\right)\right)\right). \quad (11)$$

one can write

$$P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N} y_i\left(\sum_{j=i-\lambda_i+1}^{i} g_j+\gamma_{i-\lambda_i}\right)>C\right)\le e^{-\frac{sNC}{K}}\prod_{i=1}^{N}e^{\beta_i(s,V_i)}=e^{\sum_{i=1}^{N}\beta_i(s,V_i)-\frac{sNC}{K}}. \quad (12)$$

Comparing the bound with $1-e^{-\alpha}$, we obtain

$$e^{\sum_{i=1}^{N}\beta_i(\hat{s},V_i)-\frac{\hat{s}NC}{K}}=1-e^{-\alpha}, \quad (13)$$

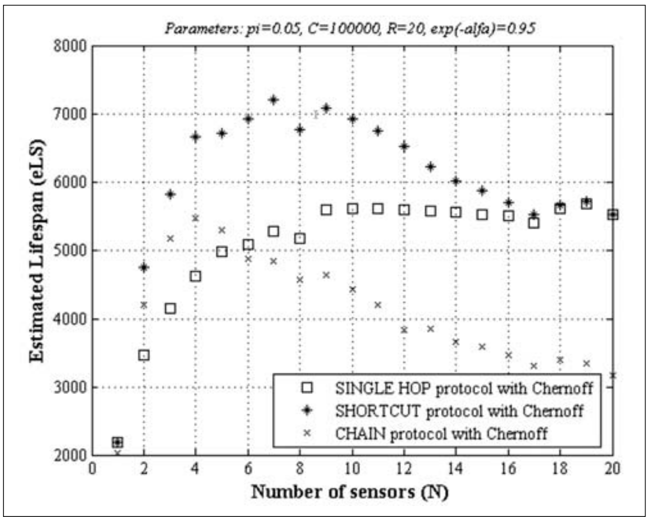
where

$$\hat{s}:\min_s\sum_{i=1}^{N}\beta_i\left(s,V_i\right)-\frac{sNC}{K}.$$

The lifespan is the solution of the following equation:

$$\tilde{K}:\sum_{i=1}^{N}\beta_i\left(\hat{s},V_i\right)=\frac{\hat{s}NC}{K}+\log\left(1-e^{-\alpha}\right). \quad (14)$$

Figure 2.
Estimated lifespan in the function of the number of sensors



As one can see the equation above determines the lifespan as a function of vector **a**, the components of which represent the probabilities of shortcut on a given node. This relationship is denoted by $\tilde{K}=\Psi(\mathbf{a})$.

Using equations (11) and (14) to evaluate $\Psi(\mathbf{a})$ for a given **a** vector, protocol optimization can take place by searching in the space of **a**-vectors to find the optimal shortcut probabilities. This can be done by gradient descent type of optimization given as follows (15):

$$a_i(n+1)=a_i(n)-\Delta\mathrm{sgn}\left\{\frac{\Psi\left(\mathbf{a}(n)\right)-\Psi\left(\mathbf{a}(n-1)\right)}{a_i(n)-a_i(n-1)}\right\}, i=1,\ldots,N$$

As a result, protocol optimization has been carried out in the following steps:

Given: N - number of nodes,
p - packet generate probability vector,
g - energy vector,
c - initial battery power vector;

Step 1. select an initial $\mathbf{a}(0)$ shortcut probability vector;

Step 2. evaluate the value of $\tilde{K}=\Psi\left(\mathbf{a}\right)$ by solving the equation $\tilde{K}:\sum_{i=1}^{N}\beta_i\left(\hat{s},V_i\right)=\frac{\hat{s}NC}{K}+\log\left(1-e^{-\alpha}\right)$;

Step 3. Perform the gradient search

$$a_i(n+1)=a_i(n)-\Delta\mathrm{sgn}\left\{\frac{\Psi\left(\mathbf{a}(n)\right)-\Psi\left(\mathbf{a}(n-1)\right)}{a_i(n)-a_i(n-1)}\right\},$$

where $i=1,\ldots,N$;

Step 4. Check the stopping criterion (i.e. $\left\|\mathbf{a}(n+1)-\mathbf{a}(n)\right\|\le\varepsilon$) and go back to Step 2. if it is not met.

In the case of single-hop protocol we have $a_i=1$, $i=1\ldots N$. Thus,

$$\tilde{K}:P\left(\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N}y_i(k)\gamma_i<C\right)=e^{-\alpha} \quad (16)$$

which leads to the following life span

$$\tilde{K}=\frac{\hat{s}NC}{\sum_{i=1}^{N}\mu_i\left(\hat{s},\gamma_i\right)+\log\left(1-e^{-\alpha}\right)}, \quad (17)$$

where $\mu_i\left(s,\gamma_i\right):=\log\left(\mathrm{E}\left[e^{sy_i\gamma_i}\right]\right)=\log\left(1-p_i+p_ie^{s\gamma_i}\right)$

is the energy required by the shortcut.

3.1. Performance analysis and numerical results

In this section a detailed performance analysis is given using the chain, the shortcut and the single-hop protocols. The aim is to evaluate the lifespan of a sensor network containing N number of sensors placed in an equidistant manner. *Figure 2* shows how the lifespan changes as the function of the number of nodes (N) in the case of the three methods described above. The distance between the base station and the farthest node was 20 meters and the nodes were located randomly subject to a Poissonian distribution.

One can see that there is a maximum lifespan in the cases of chain and random shortcut protocols with the optimal number of nodes $N_{Chain}=4$ and $N_{Shortcut}=7$, respectively.

Figure 2 shows that when the network is sparsely installed, both methods result in almost the same lifespan, while departing form the optimal number of nodes (either decreasing or increasing the number of sensors), the shortcut model definitely gives much higher relative lifespan (it is more than 37% in the case of N=7).

Figure 3 demonstrates the accuracy of lifespan estimation at the different protocols. One can see that the Chernoff bound yields a relatively sharp estimation.

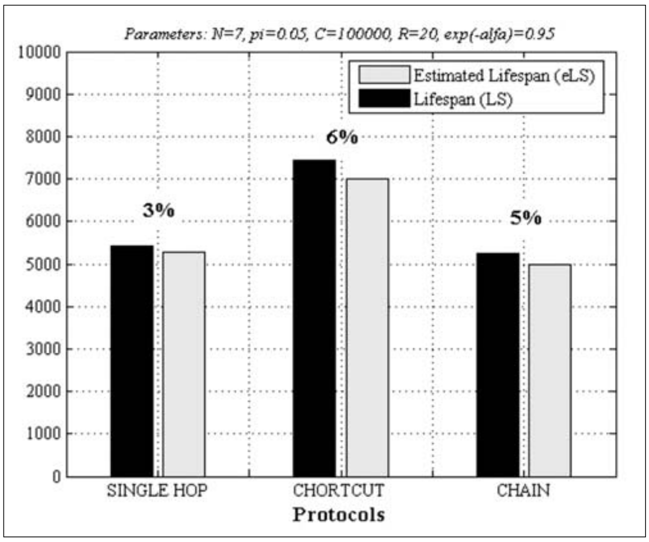


Figure 3. Lifespan and estimated lifespan values achieved by different protocols

4. Spanning tree design for optimal clusterhead selection

In this case, the network elects a CH (being the root of the spanning tree), the index of which is denoted by ξ. The nodes where packets are generated communicate with the CH via a single hop communication and then the CH re-transmits these packets to the BS. This gives rise to the following model:

- the WSN is in an energy state $\mathbf{c}(k)=\left(c_1(k),...,c_N(k)\right)$ where $c_i(k)$ denotes the available energy at node i
- a traffic vector $\mathbf{y}$ occurs with probability $p(\mathbf{y})=\prod_{i=1}^{N} p_i^{y_i}\left(1-p_i\right)^{1-y_i}$
- the energy consumption of conveying the traffic to the BS on node i is given as
$$\eta_i=\begin{cases} G_{i\xi} & \text{if } i\neq\xi \\ w\left(\mathbf{y}(k)\right)G_{\xi 0} & \text{if } i=\xi \end{cases}$$
- the new energy state is $\mathbf{c}(k+1)=\left(c_1(k+1),...,c_N(k+1)\right)$ where $c_i(k+1)=c_i(k)-\eta_i$

In order to maximize the lifespan, we are concerned with finding $\xi_{opt}:\max_{\xi}\min_{i} c_i(k+1)$, which guarantees the longest lifespan of the bottleneck node. We will refer to this objective as $\xi_{opt}:\max_{\xi}\psi(\xi)$ where $\psi(\xi):=\min_{i} c_i(k+1)$.

If vector $\mathbf{y}(k)$ is known, then this optimum can easily be calculated in polynomial time, due to the single hop nature of the spanning tree. In this case, one can evaluate the function $\psi\left(\xi,\mathbf{y}(k)\right):=\min_{i} c_i(k+1)$ "$w\left(\mathbf{y}(k)\right)$-times"

by placing the root node in different positions and ξ_{opt} can be selected.

However, this calculation cannot be carried out, as neither the BS nor the nodes are aware of the current traffic vector $\mathbf{y}(k)$.

Thus location of the root node can only be optimized in the mean sense, by finding:

$$\xi_{opt}:\max_{\xi}\sum_{\mathbf{y}\in\{0,1\}^N}\psi(\xi,\mathbf{y})p(\mathbf{y})$$

This optimization can be reformulated as follows:

$$\xi_{opt} : \max_{\xi} f(\xi) \text{ where } f(\xi) := \sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y}) p(\mathbf{y}) . \quad (18)$$

Of course the optimal index ξ_{opt} depends on the current energy state. Thus this optimization must be carried out each time when transmitting a traffic vector $\mathbf{y}(k)$ to the BS.

To carry out this optimization, one needs an efficient estimation of

$$g(\xi) \approx f(\xi) := \sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y}) p(\mathbf{y})$$

in order to circumvent the exponentially large summation in (18).

In the next section we develop a powerful approximation of $f(\xi)$ based on the modification of the Li-Silvester bounds (the original LS bound can be found in [10]).

4.1. Modified LS bounds to estimate the minimum remaining energy

In this section we propose a novel approach to estimate $f(\xi) := \sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y}) p(\mathbf{y})$ which is an improvement of the LS bounds. The original bounds are given as follows:

Let us assume that the values of $\psi(\xi,\mathbf{y})$ can be lower and upper bounded as

$0 \le \psi(\xi,\mathbf{y}) \le \psi_{\max}(\xi)$

And we pick the first K relevant vectors $Y_1 := \{\mathbf{y}^{(1)},...,\mathbf{y}^{(K)}\}$ for which $p(\mathbf{y}^{(1)}) \ge .... \ge p(\mathbf{y}^{(K)})$ and $p(\mathbf{y}^{(K)}) > p(\mathbf{y})\ \forall \mathbf{y} \notin Y_1$. Then

$$\sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) \le \sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y})p(\mathbf{y}) = \sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) + \sum_{\mathbf{y}\in Y_2} \psi(\xi,\mathbf{y})p(\mathbf{y}) \le \sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) + \psi_{\max}(\xi) P(Y_2),$$

where $Y_1 \bigcup Y_2 := \{0,1\}^N$

The estimation error can be upper bounded with $\psi_{\max}(\xi)P(Y_2)$ which is minimal due to the fact that set Y_1 contains the most probable elements.

The bound given above can be modified as follows:

Let us define the binary vector $\tilde{\mathbf{y}}$ as a "descendant" of binary vector $\mathbf{y}$ if (i) $w(\tilde{\mathbf{y}}) > w(\mathbf{y})$ and (ii) $y_i = 1$ implies that $\tilde{y}_i = 1$ (e.g. one of the descendants of vector $\mathbf{y} = (010010)$ is $\mathbf{y} = (111010)$). In the forthcoming discussion this relationship is denoted as $\mathbf{y} \prec \tilde{\mathbf{y}}$. It is noteworthy that if $\mathbf{y} \prec \tilde{\mathbf{y}}$ then $\psi(\xi,\mathbf{y}) \ge \psi(\xi,\tilde{\mathbf{y}})$ (i.e. in the case of a descendant vector there are some additional packets to be transmitted to the BS).

Let us divide the set $\{0.1\}^N$ into three disjoint subsets as follows:

$\{0.1\}^N = Y_1 \bigcup Y_2 \bigcup Y_3$ in such a manner that $Y_1 := \{\mathbf{y}^{(1)},...,\mathbf{y}^{(K)}\}$, $p(\mathbf{y}^{(1)}) \ge .... \ge p(\mathbf{y}^{(K)})$, $p(\mathbf{y}^{(K)}) > p(\mathbf{y})\ \forall \mathbf{y} \notin Y_1$, $Y_2 = \{\tilde{\mathbf{y}} : \tilde{\mathbf{y}} \succ \mathbf{y}, \mathbf{y} \in Y_1\}$ and $Y_3 = \{\hat{\mathbf{y}} : \hat{\mathbf{y}} \succ \tilde{\mathbf{y}}, \tilde{\mathbf{y}} \in Y_2\}$.

Let us define $A_{\tilde{\mathbf{y}}} := \{\mathbf{y} : \mathbf{y} \prec \tilde{\mathbf{y}}, \mathbf{y} \in Y_1\}$, $\alpha(\tilde{\mathbf{y}}) := \mathbf{y}^* : \min_{\mathbf{y}\in A_{\tilde{\mathbf{y}}}} \psi(\xi,\mathbf{y})$ and $B_{\mathbf{y}^*} := \{\tilde{\mathbf{y}} : \alpha(\tilde{\mathbf{y}}) = \mathbf{y}^*\}$.

In a similar fashion $A_{\hat{\mathbf{y}}} := \{\mathbf{y} : \mathbf{y} \prec \hat{\mathbf{y}}, \mathbf{y} \in Y_1\}$, $\beta(\hat{\mathbf{y}}) := \mathbf{y}^{**}$ is selected to be the first element in $A_{\hat{\mathbf{y}}}$ and $B_{\mathbf{y}^{**}} := \{\hat{\mathbf{y}} : \beta(\hat{\mathbf{y}}) = \mathbf{y}^{**}\}$. One must note that while $\alpha(\tilde{\mathbf{y}}) := \mathbf{y}^* : \min_{\mathbf{y}\in A_{\tilde{\mathbf{y}}}} \psi(\xi,\mathbf{y})$ involves an optimization over a smaller set, $\beta(\hat{\mathbf{y}}) := \mathbf{y}^{**}$ can be obtained automatically. As a result, the size of Y_2 in the partition of $\{0.1\}^N = Y_1 \cup Y_2 \cup Y_3$ is determined by the available time and computational resources to carry out $\alpha(\tilde{\mathbf{y}}) := \mathbf{y}^* : \min_{\mathbf{y}\in A_{\tilde{\mathbf{y}}}} \psi(\xi, \mathbf{y})$ for each $\tilde{\mathbf{y}} \in Y_2$.

Then $\sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y})p(\mathbf{y}) = \sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) + \sum_{\tilde{\mathbf{y}}\in Y_2} \psi(\xi,\tilde{\mathbf{y}})p(\tilde{\mathbf{y}}) + \sum_{\hat{\mathbf{y}}\in Y_3} \psi(\xi,\hat{\mathbf{y}})p(\hat{\mathbf{y}})$, where the second term and third terms can be upper bounded as follows:

$$\sum_{\tilde{\mathbf{y}}\in Y_2} \psi(\xi,\tilde{\mathbf{y}})p(\tilde{\mathbf{y}}) \le \sum_{\tilde{\mathbf{y}}\in Y_2} \psi(\xi,\mathbf{y}^*)p(\tilde{\mathbf{y}}) = \sum_{\mathbf{y}^*} \psi(\xi,\mathbf{y}^*)p(B_{\mathbf{y}^*})$$

$$\sum_{\hat{\mathbf{y}}\in Y_3} \psi(\xi,\tilde{\mathbf{y}})p(\tilde{\mathbf{y}}) \le \sum_{\hat{\mathbf{y}}\in Y_3} \psi(\xi,\mathbf{y}^{**})p(\hat{\mathbf{y}}) = \sum_{\mathbf{y}^{**}} \psi(\xi,\mathbf{y}^{**})p(B_{\mathbf{y}^{**}}).$$

In this way a new upper bound can be obtained as

$$\sum_{\mathbf{y}\in\{0,1\}^N} \psi(\xi,\mathbf{y})p(\mathbf{y}) \le \sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) + \sum_{\mathbf{y}^*} \psi(\xi,\mathbf{y}^*)p(B_{\mathbf{y}^*}) + \sum_{\mathbf{y}^{**}} \psi(\xi,\mathbf{y}^{**})p(B_{\mathbf{y}^{**}}). \tag{19}$$

This upper bound is sharper than the original LS bound due to the fact that

$$\psi(\xi,\mathbf{y}^{**}) \le \psi_{\max}(\xi),\ \psi(\xi,\mathbf{y}^*) \le \psi_{\max}(\xi) \text{ and } \sum_{\mathbf{y}^*} p(B_{\mathbf{y}^*}) = P(Y_2),\ \sum_{\mathbf{y}^{**}} p(B_{\mathbf{y}^{**}}) = P(Y_3).$$

For the sake of notational simplicity, the bound in (19) will be denoted as

$$g(\xi) := \sum_{\mathbf{y}\in Y_1} \psi(\xi,\mathbf{y})p(\mathbf{y}) + \sum_{\mathbf{y}^*} \psi(\xi,\mathbf{y}^*)p(B_{\mathbf{y}^*}) + \sum_{\mathbf{y}^{**}} \psi(\xi,\mathbf{y}^{**})p(B_{\mathbf{y}^{**}}).$$

4.2. Computational model to find the optimal CH

Based on the discussion above, the modified LS bound will be used to estimate $f(\xi)$, which gives rise to the following two protocol optimization models.

In the first case, we assume that $\mathbf{y}(k)$ is known prior to the transmission and then optimization is carried out as indicated by *Figure 4*.

However, this case will only serve as a reference for the performance analysis, as the traffic vector $\mathbf{y}(k)$ cannot be known prior to the transmission.

Therefore, the protocol optimization takes place according to (18), which yields the algorithm depicted in *Figure 5*.

4.3. Numerical results

In *Figure 6* the lifespan obtained by the chain, single-hop, and CH protocols are plotted as a function of the number of the nodes. The results were obtained on the same WSN as described in Section 3.1.

In the case of adaptive CH protocol, the clusterhead is selected as a function of the current traffic vector $\mathbf{y}(k)$, whereas in the case of average CH protocol the clusterhead was selected by maximizing the expected value of the minimum remaining energy and the expectation was taken over the whole traffic state space $\{-1,1\}^N$. On the other hand, the chain protocol forwarded packets node-by-node to the BS, while the single hop protocol transmitted packets directly to the BS from each active nodes.

Figure 4. Adaptive CH optimization assuming known traffic vector $\mathbf{y}(k)$

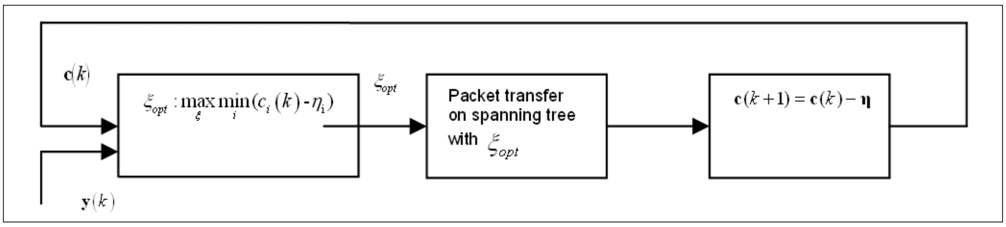
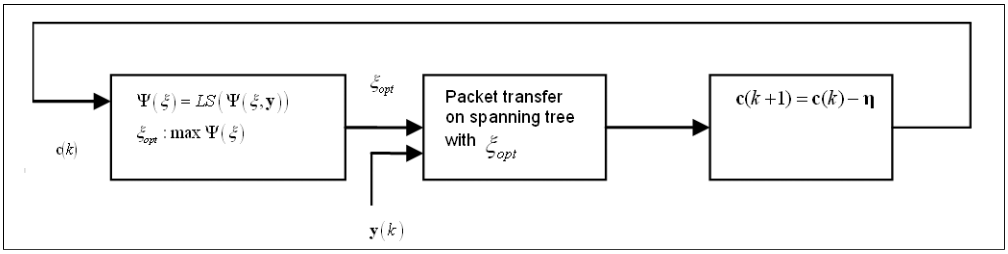


Figure 5. CH optimization on the basis of energy vector $\mathbf{c}(k)$



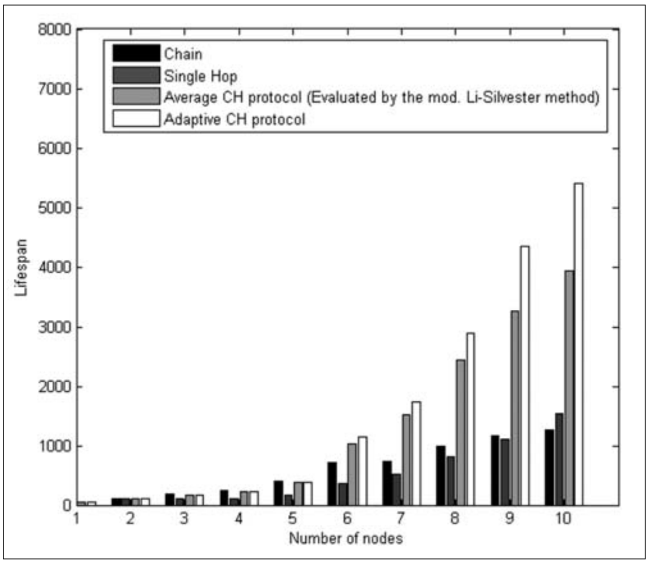


Figure 6.
Comparing the lifespan of CH type protocols to the traditional ones

One can see that the CH type protocols outperform the chain and single-hop communication as far as the lifespan is concerned. In the case of ten nodes the lifespan has been increased to three or four times longer than the lifespan obtained by traditional methods. This strongly motivates the use of CH type protocols.

5. Conclusions

In this paper, energy balancing of WSN has been studied by statistical tools. A novel "random shortcut" protocol has been introduced and the optimal probability distribution for selecting destination for packet forwarding has been found.

Identifying the optimal CH has also been considered by using a spanning tree model and a novel bound to estimate the means of the remaining energy function. Both protocols can significantly increase the lifespan of WSN. The performance of the methods have been tested by extensive simulations which also demonstrated the improvement on the lifespan.

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