Numerical analysis of mobility management algorithms

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This paper investigates mobility management strategies from the point of view of their need of signaling and processing resources on the backbone network and load on the air interface. A method is proposed to model the serving network and mobile node mobility in order to be able to model the different types of mobility management algorithms. To obtain a good description of the network we calculate parameters from given topologies that we think are the most important ones. Mobility approaches derived from existing protocols and other possible mobility scenarios are analyzed and their performance is numerically compared in various network and mobility scenarios. The aim is to give general design guidelines for the next generation mobility managements on given network and mobility properties.

1. Introduction

Information mobility has become one of the most common services in the modern world with the penetration of the portable phones and other mobile equipments. The wireless multimedia and other services have many requirements and the resources in the serving network are often expensive and limited.

In the first mobility protocol designs the main scope was to create a well-functioning mobility. For example the Global System for Mobile Communication (GSM) network uses a cellular approach to save bandwidth on the air interface but does not really focus on the problem of signaling load on the wired serving network. In the Mobile IP (MIP) structure the IP mobility is in the main scope. There are many enhancements of MIP to optimize the original protocol and introduces for example hierarchy, location tracking to obtain a cheaper solution. However, Host Identity Protocol (HIP) is drastically different from MIP: their mobility approaches are similar but implemented on different network layer levels. Wireless Local Area Networks (WLAN) are constructed like the original Local Area Networks (LAN) and provide mobility only within the radio interface and use Dynamic Host Configuration Protocol (DHCP). Future protocols might use different media and technological background to provide mobility. For this reason it is appropriate to treat mobility as an abstract problem regardless of actual technical solutions.

The advantage of our work is that we do not focus on a selected technology – not even on a given network generation – but discuss mobility in general within the modern computer and telecommunication networking technologies. We compare selected mobility approaches and show how the network properties affect the usability of each. The aim is to find the suitable one for different scenarios or at least to give guidelines how to construct the network for a protocol or adjust the protocol to the network.

2. Abstract mobility management

In this paper, the mobility management is discussed generally regardless of the very technology used. One will see that the approaches discussed here could be applied for various types of mobility management protocols on different technology levels. We try to grab the most significant properties of the mobility that is worth to discuss within the scope of the modern mobility protocols.

We define *Mobility Management System* as an application running on network nodes that helps to locate the mobile equipment towards its unique identifier.

- *Mobile Nodes* (MN) are the mobile equipments who want to communicate to any other mobile or fixed partner.
- There are *Mobility Access Points* (MAP) as the only entities that are capable to communicate with the Mobile Equipments.
  (Note: mobility does not necessary imply radio communication. It means only that the Mobile Node changes its Mobility Access Points and when it is attached to one, communication between them can be established.)
- *Mobility Agents* (MA) are network entities running the mobility management application.
- There is a core network that provides communication between the Mobility Access Points and has a structure that can be described with a graph.
  Vertices are either Mobility Access Points or Mobility Agents or other serving nodes who are not part of the mobility management application and the edges can be any kind of links (even radio links) for the data communication between the vertices.

With this definition, one can see that most of the functionalities of the current mobility protocols and others under development can be generally described.

However, this model is too general and we should restrict the discussion with some practical assumptions:

- A Mobility Access Point is always a Mobility Agent. (In our discussion, there can always be a sub-network of multiple physical access points under a single Mobility Access Point. We do not discuss the lowest (micro) level of mobility.)
- Mobile equipment can communicate with multiple Access Points at a time but one connection is necessary and enough to maintain the correct communication. The mobile can also attach and detach from any Mobility Access Points.
  At this point we assume that the mobile node is administrated only at one agent.
  This means that the problem of finding the mobile node is the same as to find the correct agent.
- The nodes in the core network communicate and find each other with a given protocol or method (for example via IP routing). For this reason this part of the mobility protocols is not discussed.

Now mobility management is simplified to a protocol that finds the correct, marked Mobility Access Point where the MN is attached. This suits to our aim to investigate the properties of various management strategy approaches since the number of messages sent and the number of tasks completed can be calculated. With cost parameters one will be able to adopt the model to exact solutions and can analyze them.

We derived the strategies into the *Centralized-, Hierarchical-, Tracking- and Cellular-like* approaches. There can be some other special approaches but mostly they can be classified into these categories. It is also common that the mixture of applications is used on different mobility layers. By our investigations we believe that design guidelines for new generation network mobility protocols can be given.

3. Network graph and node mobility parameters

In this section, we introduce how we will handle the networks on which the mobility management algorithms work. To derive the main parameters we will have to model the behavior of the mobile nodes first. There will be general and algorithm-specific parameters introduced.

Secondly, the three cost dimensions we want to handle in this paper will be introduced. These are the “signaling on the links” (C^{signal}) as a bandwidth and interworking equipment usage, the “processing in the nodes” ($C^{process}$) which are taken into account on the nodes running the mobility protocol, the “air interface usage” (C^{air}) containing explicitly battery consuming as well.

3.1. Modeling the network

In many works the network is modeled in order to emphasize the properties of a single protocol compared to another one. This approach has the disadvantage of inflexibility since new protocols can not be included in the comparison and also it is difficult to follow little modifications in the protocols.

An approach of the network modeling uses the given network structure that is essential to make an appropriate examination in those cases but limits the scope of discussion. For example, when a GSM cell structure is used, no vertical handovers are taken into account: another mobility protocol might have a different structure to cover the same geographical region. One can see that in these cases, the graph, describing the network might not be drawn on a plane.

For this kind of reasons, many works describe a network using single parameters, for example by a general average distance between nodes. With this approach, any kind of network could be described. However, introducing these parameters is not enough to compare most of the protocols.

Summing up the requirements, we introduce a method to model the given networks to get the benefits of the first approach and we provide a method how the protocol-specific parameters and also additional ones can be derived in order to generalize the discussion just like in the case of the second approach.

3.2. Deriving parameters of a given network

Let us have a given network topology with a given MN behavior. The network is modeled with a graph just like the possible movements of the mobile. The behavior of the mobile node that is the frequency of some kind of a handover between two mobility access points will be modeled with Poisson processes like in [5].

Let us assume that the behavior of the MN can be modeled with a Markov chain, given with a rate matrix. In this matrix, all the possible (in practice: the practically possible) MAs are listed where the Mobility application runs. (These MAs can also denote single access points, bigger networks or the Home Agent if desired.)

The number of MAs is n and so the matrix will be an $n \times n$ matrix where each element denotes how frequent the movement of the mobile is from MAPi→MAPj. (If an MA is not a MAP then there are 0 values in its row and column.) From the rate matrix the transition matrix can be determined easily. We assume that the matrix, without the non-MAP nodes, is practically irreducible and aperiodic that implies that the chain is stable and there exists a stationary distribution. This will be denoted by a density vector. In this vector, the ith element denotes the probability of the MN being located under the ith MAP. (For MA nodes that does not support access point functionality, there exists an element in the vector with 0 value.)

Let us have the corresponding network graph given with its adjacency matrix A. This matrix should include all the nodes in the network where the mobility application runs (all the MAs again) so has the same $n \times n$ size as matrix. With the Floyd algorithm the optimal distances between the nodes can be calculated (even with weighted or directed edges as well). The distance between nodes will be the sum of weights on the shortest path from one to the other. Let this result matrix be given. In the *i*th row of the matrix, the distances from FAi are listed. Let the distances from the HA, – a special FA – be given with the vector *a*.

We will have parameter *w* to denote the average of the weights in the network. It can be calculated by summing up the elements of the matrix and dividing it by n^2.

3.2.1. Determining m

Parameter *m* will denote the average depth level, that is the average number of edges on the shortest path from the MN to the HA. Clearly, the average number of vertices among the path is $m+1$. We will use matrix A_d and vector $\underline{a}$ to calculate this parameter. Both have to be normalized with the average weight of edges in the network (*w*). Now *mw* can be calculated by determining the weighted average of the distances where the weights are the probabilities that the node is under a given MAP:

$$m = \frac{\underline{a} * \underline{b}}{w} \tag{1}$$

where * stand for the scalar product. One can see that the nodes which are not MAPs have a 0 multiplier and do not count in the average distance as expected.

3.2.2. Parameter g_T

We will have another parameter like *m* that is the average distance between two nodes who handle the MNs handovers. They might be connected, but they can also be quite far from each other logically due to different technologies especially in the case of vertical handovers. So as we see this parameter has to denote the weighted average value of the length between every two neighboring MAs where the mobile can attach. Then it is calculated as follows:

$$g_T = \frac{\underline{b} * tr(A_d \cdot B_\Pi)}{w} \tag{2}$$

Our notation indicates that this parameter will have the most effect on the Tracking-like management solutions as we will see.

3.2.3. Parameter g_H

This parameter denotes how far is the nearest hierarchical junction to register at in the average, if we consider the optimal covering tree of the network with the HA in the root. The junction node is the nearest common node of the paths from HA to the old and the new FA of the MN. (In most cases, it is not possible to achieve the optimal tree structure since the different service providers will not mesh their networks: approximate values can be used instead.) About determining of parameter g_H can be read in [6].

3.2.4. Parameter g_C

This parameter will denote the average distance of MAPs from the main MA of a Location Area in the Cellular-like approaches. It is an NP full problem to calculate the optimal cell structure, but there are algorithms approximating it very well in some sense. We have run the algorithms developed and published in [12].

3.3. Modeling the mobile node

As we have seen, matrix B_Q describes the movement behavior of the MN, handover-wise. Summing up the *i*th row in this matrix we get a rate of how frequently the MN moves from the *i*th MA (MAP) with a Poisson-process. Let λ denote the average parameter of the Poisson-process (at each MAP) and so denote the rate of handovers for a general MN anywhere in the network.

The other parameter that can be introduced in a similar manner is the rate of receiving a call: μ. This parameter can also be time- or location-dependent. We take its average value like we did it in the case of λ and we assume it is constant in the examined very small time interval just like we did in the case of matrix B_Q and through the whole modeling.

Using the achievements in [6], let us introduce ρ as the "mobility ratio" meaning the probability that the MN changes its FA before a call arrives:

$$\rho = \frac{\lambda}{\lambda + \mu} \tag{3}$$

3.4. Definitions of cost constants

The three main classes of cost types will be introduced here. One will see in Section 4 that modifying the ratio of some parameters (for example the registration and packet forwarding costs) will have strong effect on the results.

If one tries to design a mobility management algorithm and also wants to implement and use it he has to decide the network level he wants to use. Also the equipments might be different. It is possible to modify the parameters we will introduce and then to have a relevant calculation on the expected costs.

3.4.1. Link related constants

cu: The unit cost of one update on a link.
cd: The unit cost of one delivery on a link.

3.4.2. Node related constants

cr: Registration cost, as the cost of the process in the MAP that has to run in the case when a MN node wants to attach. This can include the generation cost of a temporary ID, database handling, agent discovery etc.
cf: Forwarding cost at a MA. If a signaling message reaches a MA it has to decide if there is some process has to be executed with the package and where to forward it. (This can be really low for a number of protocols but also high as well.)
cm: This is the constant cost of modifying some node related records in a MA.
cec: The cost of building up a message. For example to encapsulate a message when a corresponding node wants to communicate with the MN in the MIP structure.
crc: The cost of recapsulating or rebuilding a message.
cdc: The cost of decapsulate or open the message at an endpoint.

3.4.3. Mobil equipment connection related constants

cau: The cost of uplink messaging between the MN and the MAP.

cad: The cost of downlink messaging between the MN and the MAP.

4. Modeling the existing approaches

In this section, the selected five main types of mobility management protocols are described and modeled with their signaling-, processing-, and air interface cost functions [1]. One will see that these main protocols could be applied to most of the existing mobility approaches.

4.1. Centralized approaches

In this management structure the mobile always sends location update messages in case of handover to a centralized management node, which maintains a database to contain the location of Mobile Nodes. Because of this the central agent is always able to forward the packets to the MN (Mobile IP [10]), or to send back the reachability of the MN (SIP).

$$\begin{aligned} C_{CENT}^{signal} &= \rho m c_u + (1-\rho) m c_d \\ C_{CENT}^{process} &= \rho(c_r + (m-1)c_f + c_m) + (1-\rho)(c_{ec} + (m-2)c_f + c_{dc}) \qquad (4) \\ C_{CENT}^{air} &= \rho c_{au} + (1-\rho) c_{ad} \end{aligned}$$

One can see that the cost functions are obvious and simple. The second main advantage of this protocol is its simplicity: these approaches can be installed by setting up a Central Agent in the network and by running an IP-level software module on the MN. There is no need to change any other entity in the network, therefore it is cheap and easy to install.

On the other hand, centralized mobility puts extraordinary high overload on the bearer network and uses non-optimal routing, which is unacceptable. However this solution is far from the optimal, still, most of the mobility implementations use the same kind of this centralized approach.

4.2. Hierarchical solutions

Instead of the global management node regional management system can be used to reduce the signaling traffic by maintaining the location information locally. For this reason we can use the MAPs and MAs as local agents that have database to store the actual IP addresses of MN. So we can consider this hierarchical network structure as a tree of MAP, MA and other network node with Central Agent in the root of the tree.

Because the location information is sent only to the nearest MA, the costs function changes compared to the centralized solution. The advantage of this method is the more optimal functionality, and smaller load on the bearer network. However the change of some other entity is needed in the network, therefore the solution is more expensive.

An example for such solution is the Hierarchical Mobile IP (HMIP) [4]:

$$\begin{aligned} C_{HIERARCH}^{signal} &= \rho g_H c_u + (1-\rho) m c_d \\ C_{HIERARCH}^{process} &= \rho(c_r + (g_H - 1)c_f + c_m) + \qquad (5) \\ &\quad + (1-\rho)(c_{ec} + (m - g_H - 1)c_f + c_{rc} + (g_H - 1)c_f \\ C_{HIERARCH}^{air} &= \rho c_{au} + (1-\rho) c_{ad} \end{aligned}$$

4.3. Cellular-like solutions

For mobility problem there are cellular-like solutions as well, whose idea comes from the GSM protocol.

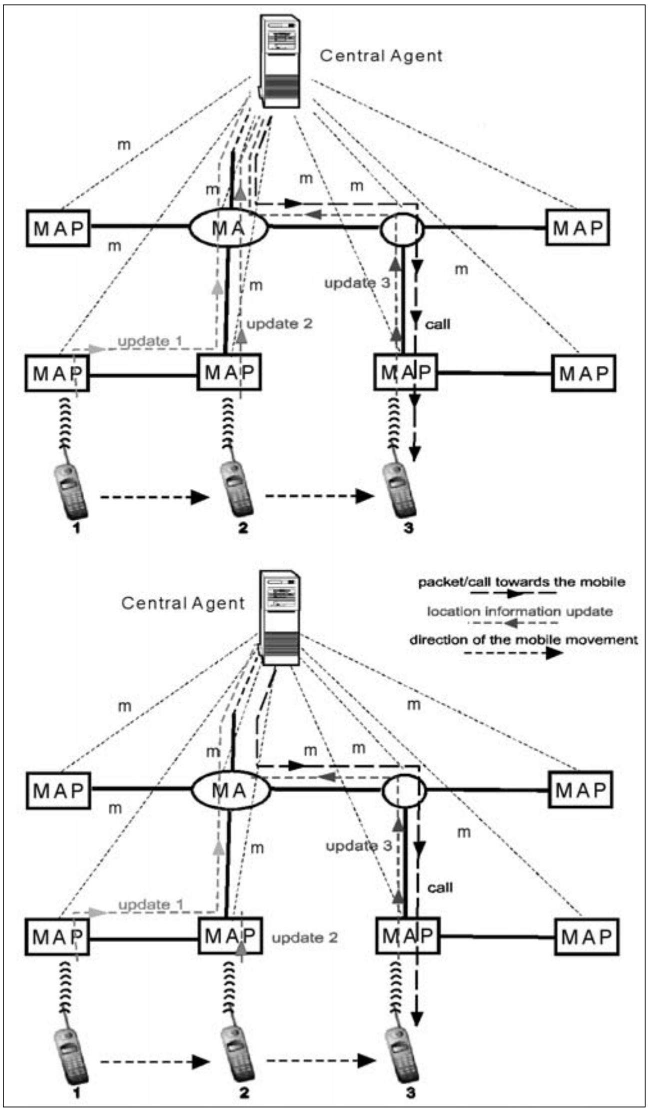
The advantages of these approaches are the quick handover mechanism in lower layer and cheap passive connectivity as it can be seen through the cost functions in *Figure 1* as well. The disadvantage is that the building of the network has to be done carefully and too many paging messages will cause an extreme increase in the costs. In cellular like solutions two constants related to the network topology are very important:

nc: The average number of MAPs in a page.

nd: The number of pages in the whole network.

Three main subtypes could be distinguished, which are introduced in the next sections.

Figure 1. Centralized and hierarchical strategies



4.3.1. Standard cellular

For mobility problem there are other cellular-like solutions. One well known example is Cellular IP (CIP) [3].

The solution builds strongly on the fact, that from the large number of mobile nodes only a small percentage is receiving data packets. For this reason we can introduce well-defined optimized areas, called paging areas, and it is enough to know in which paging area the idle mobile are moving. In this case the hop-by-hop manner routing leads the packet only to the domain border of the paging area.

From this point of the network to the mobile, the nodes in the paging area do not store any information about the idle mobiles, accordingly in case of a packet addressed to an idle mobile the paging area is flooded with the packet by broadcast message (6):

$$C_{CELLULAR}^{signal} = \rho(1-P_c)g_H c_u + (1-\rho)((m-g_C)+(n_C g_C c_d)+g_C c_u$$

$$C_{CELLULAR}^{process} = \rho((1-P_c)c_r + g_H c_f + c_m) +$$
$$+(1-\rho)(c_{ec}+(m-g_C-1)c_f + c_{rc} + (g_C-1)n_C c_f + c_{dc}c_r)$$

$$C_{CELLULAR}^{air} = \rho((1-P_c)c_{au}) + (1-\rho)(n_C c_{ad} + c_{au})$$

where P_C the probability of entering a new page.

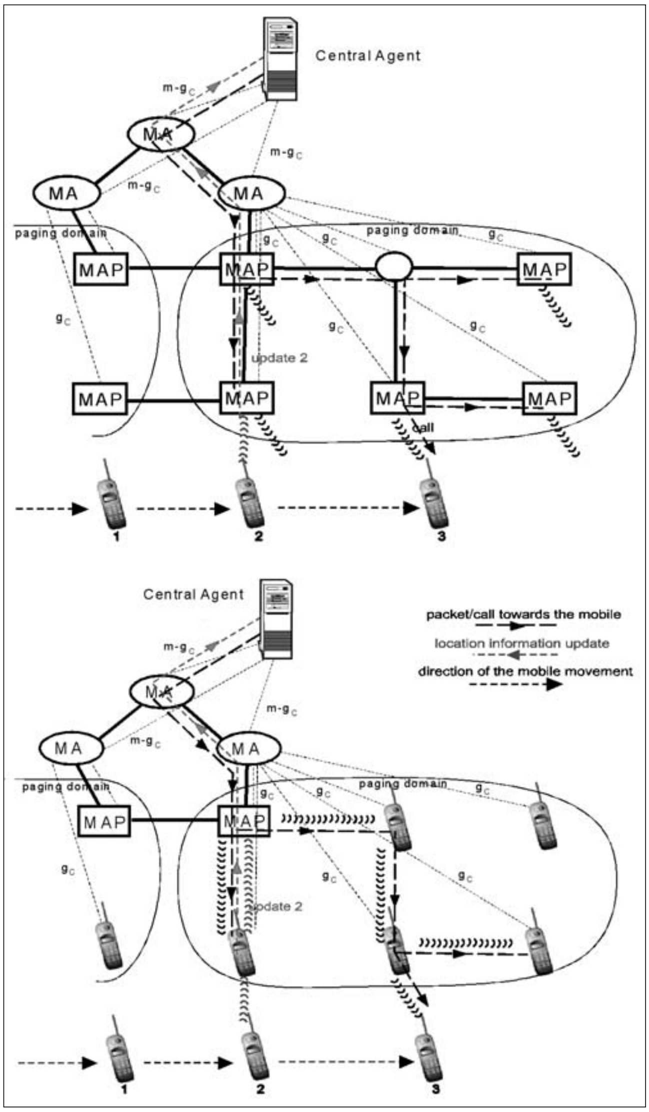


Figure 2. Standard cellular and MANET-like solutions

4.3.2. Hierarchical paging

The main idea behind the Hierarchical Paging [8] is that not only the lower layer network is flooded with the packet but broadcast message is used to find the paging controller MA in the higher layer as well. With this functionality signaling costs could be saved because update messages are not sent to HA, but only to the MA which controls the page.

But in case of calling the multilevel flooding causes high network load (7).

$$C_{HP}^{signal} = \rho(1-P_c)g_C c_u + (1-\rho)((m-g_C)+(n_C g_C c_d)+g_C c_u$$

$$C_{HP}^{process} = \rho((1-P_c)c_r + g_C c_f + c_m) +$$
$$+(1-\rho)(c_{ec}+(m-g_C-1)n_d c_f + c_{rc} + (g_C-1)n_C c_f + c_{dc}c_r)$$

$$C_{HP}^{air} = \rho((1-P_c)c_{au}) + (1-\rho)(n_C c_{ad} + c_{au})$$

4.3.3. MANET in the page areas

The MANET [9] in the page areas solutions introduced by us could be the best solution when we would like to save the infrastructure cost and the air interface using is cheaper. In this management system it is assumed that all MN could be reached via other MNs. Paging areas are defined like in other cell-like solutions, but only one MAP exists in one page, through this the packets are routed using an optimal MANET algorithm. Advantage of this solution also is that signaling cost can be saved with correct MANET protocol in a page.

However, in the suboptimal case some mobiles could not be reached, and aggregate air interface cost can be high *(Figure 2)*. (8)

$$C_{MANET}^{signal} = \rho(1-P_c)g_C c_u + (1-\rho)((m-g_C+1)+(P_M n_C g_C c_d)+g_C c_u$$

$$C_{MANET}^{process} = \rho((1-P_c)c_r + g_H c_f + c_m) +$$
$$+(1-\rho)(c_{ec}+(m-g_C-1)c_f + c_{rc} + P_M n_C g_C c_f + c_{dc}c_r)$$

$$C_{MANET}^{air} = \rho((1-P_c)(g_C-1)c_{au}) + (1-\rho)(P_M n_C g_C c_{ad} + c_{au})$$

In MANET like solutions at ad-hoc mobility level the request have to be sent via P_M percent of mobile nodes in order to be delivered it to the destination mobile node in a page.

4.4. Tracking-like Solutions

In the tracking-like approaches each mobile node has an entry in a Central Agent like in other solutions. This CA stores the address where it received location update message from. It is the address of an MAP, and is a next-hop towards the mobile node. The mobile node is either still connected to that MAP, or that MAP knows another next-hop MAP towards the mobile.

Finally the mobile node can be found at the end of a chain of MAPs. One can read more about these protocols in [2,6,11].

4.4.1. Wireless tracking

In case of tracking handover of wireless tracking the mobile sends the address of the new MAP node to the old MAP node over the air interface.

$$C_{WLESSTR}^{signal} = \rho P_H g_H c_u + (1-\rho)(g_H c_d + M[h_r]g_T c_d + (1-P_0)g_H c_u \qquad (9)$$

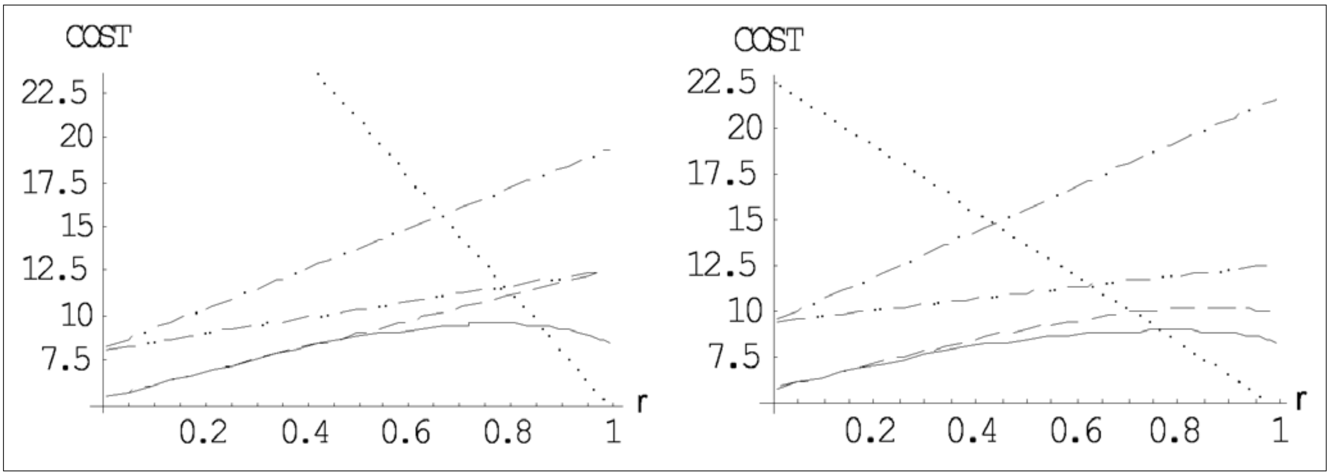


Figure 3.
One can see the summed cost functions of centralized-like (one-dot-dash), hierarchical-like (two-dot-dash), wireless (dashed) and wired (solid) tracking-like, cellular-like (dotted) approaches here with the vary of the mobility ratio. The two figures show the costs on different networks.

$$
\begin{aligned}
C_{WLESSTR}^{process} = {} & \rho((1-P_H)(c_r+c_m)+P_H(c_r+(g_H-1)c_f+c_m)+ \\
& +(1-\rho)(c_{ec}+(m-1)c_f+P_0c_{dc}+ \\
& (1-P_0)(M[h_r]((g_T-1)c_f+c_{rc})+c_{dc}+(g_H-1)c_f+c_m))
\end{aligned}
$$

$$C_{WLESSTR}^{air} = \rho c_{au}+(1-\rho)c_{ad},$$

where the $M[h_r]$ the number of tracking handovers after a normal handover, P_H the probability of that the Markovian model is in state H [6], which means a normal handover in the next step.

4.4.2. Wired tracking

Wired tracking differs from the wireless in the method of the tracking handover. In this case the MN sends the address of the new MAP node to the old MAP node through the wired network.

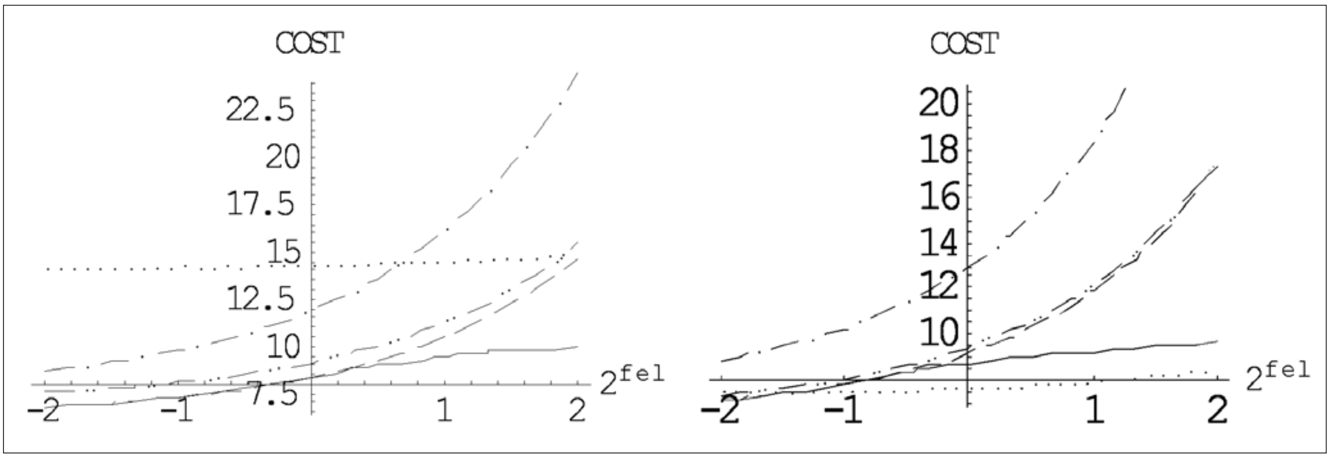
$$
\begin{aligned}
C_{WTRACK}^{signal} &= \rho(g_T(1-P_H)+g_HP_H)c_u+(1-\rho)(mc_d+M[h_r]g_Tc_d+(1-P_0)g_Hc_u \\
C_{WTRACK}^{process} &= \rho(c_r+(g_T-1)c_f+c_m)+(1-\rho)(c_{ec}+(m-1)c_f+P_0c_{dc}+ \\
& (1-P_0)(M[h_r]((g_T-1)c_f+c_{rc})+c_{dc}+(g_H-1)c_f+c_m)) \qquad (10) \\
C_{WTRACK}^{air} &= \rho c_{au}+(1-\rho)c_{ad}
\end{aligned}
$$

5. Numerical results

We do not attempt to give an exhausting numerical analysis with our method here since this paper focuses on the modeling framework itself. However, we give a very few examples for the type of investigations that could be performed using our model. The exact numerical values of the results are not important, we focus on the behavior of mobility with the change of the parameters.

In *Figure 3* one can see the difference between the approaches considering all the cost types (signaling, processing, air). It is clear that with the bigger frequency of handovers (ρ) the cost is bigger for the centralized-like, hierarchical-like and wired tracking-like approaches since each handover gives more signaling on the network. In the wireless tracking-like case if the number of handovers increases between the incoming calls, it starts saving the costs of the rerouting of the packets. In the centralized-like ones, it is clear that the rarer there is an incoming call the lower load the network has. The cost

Figure 4.
The uplink/downlink vary dependency with the same notation at ρ=0.7 and ρ=0.9.



is obviously high in this case. The same case is printed in both figures, but the values of g_T; g_C network parameters are significantly less than g_H (more meshed network). One can see that the wired tracking-like solution is getting cheaper as well and begins to behave as its tracking-like pair.

In *Figure 4* the mobility ratio is fixed to $\rho = 0.7$ and $\rho = 0.9$, respectively. On the other hand, the cost of a single upload (c_u) to a single download (c_d) is exponentially changing from the half to the twice on the horizontal axis. Most of the solutions are more expensive if the upload is higher but it can be seen that the wireless tracking cuts this cost as expected.

In *Figure 5* the different cellular strategies can be seen as a function of the air interface costs. One can see that the most optimal solution is MANET with low air costs. The larger the air interface cost is, the lower the difference is between the Hierarchical Paging diagram and Standard Cellular diagram.

One is able to perform further examinations using our Mathematica program.

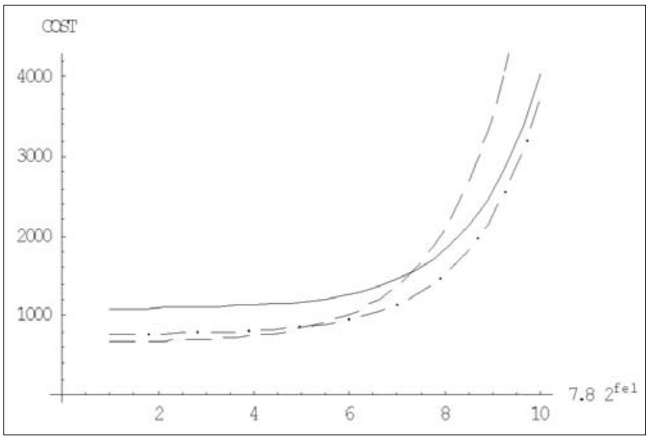


Figure 5.
Cost of cellular strategies as a function of air interface costs
(dashed: MANET, one-dot-dash: Standard Cellular, solid: Hierarchical Paging)

6. Conclusion and future work

Our primary aim was to develop an abstract modeling method for mobility managements. In this paper, we grabbed numerous significant parameters of mobility and modeled the mobile node behavior as well as the network and some general management strategies. Using our results, it can be shown which mobility management gives the best solution in different network scenarios and which aspect of resources could be a bottleneck in each case. One can use our achievements to analyze various mobility managements.

Our secondary aim, that is part of our future work, is to use the measurements to provide guidelines for the design of new mobility management algorithms and to propose solutions for different requirements.

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