Modelling packet queuing of DSL access lines for the case of complete and partial rejections

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In this paper we provide an exact data-layer model and mathematical analysis of priority queuing systems representing DSL access networks on packet level with pre-emptive option. We demonstrate the accuracy and the efficiency of our numerical analysis by presenting numerical results based on simulations and numerical analysis both for complete and partial rejections. Consequently, this analysis could be applied for an in-depth packet-level performance evaluation of recent DSL systems.

1. Introduction

Nowadays the mostly used protocols in the access network are those from the family of Digital Subscriber Line (DSL) [1,2]. A wide range of DSL technologies is available providing different sets of maximum available capacities and physical reach. From the aspect of available resources it is well-known that the edge of the network is less developed than its middle level. Therefore, the access capacity often appears to be the bottleneck of the network connection. The usual method to confront successfully this bottleneck, as also proposed by 3GPP and ITU-T, is to classify packet flows into four classes that cover applications with the same order of magnitude of Quality of Service (QoS) requirements. Packets of each class are stored in separate buffers and usually served by strict priority scheduler [3].

This paper is motivated by the performance evaluation study of DSL based access network supporting QoS. The related data-layer model leads to the analysis of priority queuing system with finite buffers and bursty arrivals, where at the inlet of a common DSL line a strict priority scheduling is applied on the fragmented upper layer data units, while depending on the actual implementation complete or partial rejection could be applied.

The study of priority queuing systems today is also an actual topic in the field of queuing research. However, the exact description of such a system is not yet available. Instead, several approximate solutions can be found in the related literature, which are not sufficient enough in practical performance analysis. The modelling approach presented in this paper overcomes the requirements of performance evaluation of both types of rejection rules.

2. DSL access architecture

Architecturally, the DSL customers are connected to the access network by using DSL modems that are aggregated into DSL Access Multiplexer (DSLAM) through DSL access lines as depicted in *Figure 1*.

The latest standards of DSL [1,2] offer two options of packet transport. In the ATM-based DSL technologies ATM Adaptation Layer 5 (AAL5) is used to encapsulate higher layer packets. A packet entering the DSL modem or DSLAM output port is first converted to an AAL5 Protocol Data Unit (PDU) then the whole PDU is segmented into ATM cells. In the other case when Ethernet-based DSL technology is considered, Packet Transfer Mode – Transmission Convergence (PTM-TC)

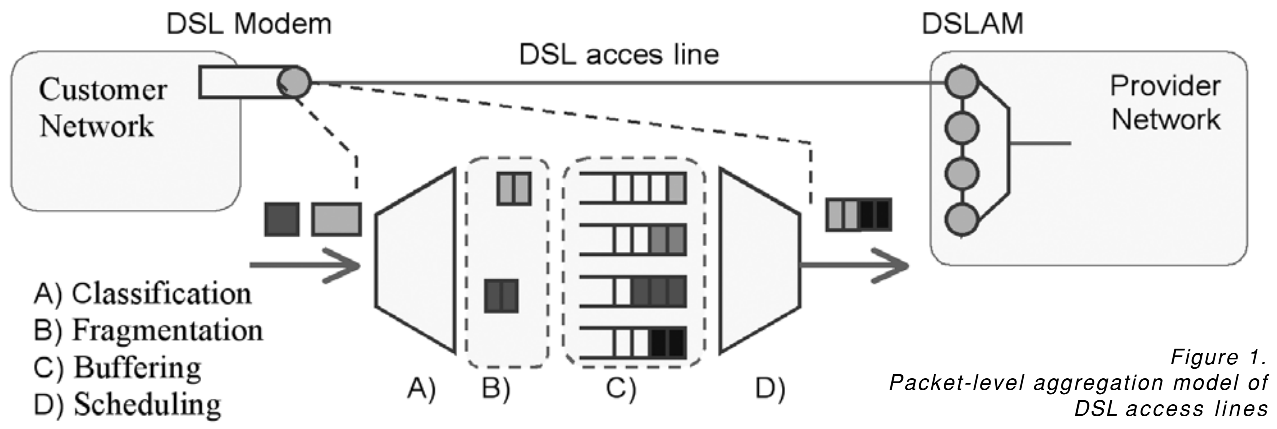


Figure 1. Packet-level aggregation model of DSL access lines

is introduced that supports transmission of higher layer packets by applying the 64/65 byte encapsulation method of High-Level Data Link Control (HDLC) framing.

At the data plane, when a packet arrives at an empty queue, even if it has higher priority than the packet has which is currently under service, it has to wait for the service completion. This mechanism is particularly problematic for low-rate transmissions. The service time of a full length Ethernet frame at the lower priority class introduces a considerable delay in the first priority queue, especially at the line rates of today's access networks. In order to reduce this kind of delay, pre-emption mechanisms such as ATM AAL5 encapsulation or PTM-TC with pre-emption option enabled, since fragmenting packets into small pieces would lower the additional delay introduced by serving these large packets from different classes.

The above described data-layer model including segmentation of user traffic and pre-emption option leads us to the model of priority queuing system with batch arrivals. Besides, when in the real system congestion occurs, at the buffer of user traffic two options are implemented. When complete rejection rule is implemented, the whole higher-layer data unit is dropped in the case of congestion, while partial rejection first fills the free slots in the buffer, and only the remaining segments are dropped. During the proposed analysis both options are considered.

3. Related work

A number of papers have been published regarding the analysis of priority queuing systems since the first initial results of Takács [12]. Although the study of priority queuing systems is also an actual topic in the field of queuing research nowadays, the exact description of such a system is not yet available. First, we summarize the works in which infinite buffers are assumed. The problem is less complicated and some nice and explicit formulae can be provided in this case. Besides, the results for systems with infinite buffers are good approximations of finite, large buffer systems in some certain conditions. These papers, e.g. [6], often apply generating functions, Laplace transform, or matrix geometric methods to determine the distribution of waiting time.

Assuming the arrival process is Poissonian, Takács [7] gave necessary and sufficient conditions for the existence of a stationary limit distribution of the waiting time. He also provided the Laplace transform and the first three moments of the limit distribution. In [8] two priority classes are considered. The arrival processes are assumed to form four mutually independent renewal processes determined by general distributions. Limit theorems are obtained for the low priority waiting time and for the total uncompleted service time of unfinished work in the system at time t.

Non-preemptive priority queues with MAP (Markovian Arrival Process) arrivals were considered in Takine's paper [9]. The service times of each priority class are i.i.d. random variables with a general distribution function. Using both the generating function technique and the matrix analytical method, they derived various formulas for the marginal queue length distribution of each priority class. Furthermore, they provided the delay cycle analysis of the waiting time distribution of each class and characterized its Laplace-Stieltjes transform.

Xue and Alfa [10] assumed BMAP (Batch MAP) arrivals of the high priority class and MAP arrivals of the low priority class in the case of two queues. A sufficient condition under which this tail probability has asymptotically geometric property was derived. If the asymptotically geometric property holds, a method was designed to compute the asymptotic decay rate. Alfa, Liu and He [11] used the matrix geometric method to study the MAP/PH/1 general pre-emptive priority queue with multiple classes of jobs. They determined the stationary behaviour of the system. Next, the distribution of the number of waiting packets and their waiting time are easily calculated.

Reducing the amount of the necessary computation is the goal of the work of Van der Heijden et al. [12]. The idea of their approximation method for N classes of customers was the following: for each class, aggregate the remaining customers into one class and evaluate the performance of the system with these two classes. This method leads to the analysis of N two-class systems instead of the analysis of one N-class system. The service time of the aggregated class is approximated by a hyper-exponential distribution.

Finally, we summarize some further works in which priority queuing system with finite buffers were analyzed. In the case of finite buffers the packets may be lost if the buffer is overloaded. Sharma and Virtamo [13] investigated a priority system with two buffers, Poisson arrivals, and general service time. An algorithm was given to calculate the distribution of the waiting time and the rejection probability. Gómez-Corral et al. [14] used a continuous-time Markov chain to describe the state of the system at arbitrary times, constituting a finite QBD process. Computationally convenient formulas were derived for various performance measures: the blocking probability, the stationary distribution of state at pre-arrival epochs, post-departure epochs, and loss epochs.

4. The queuing model

Let us consider a priority queue with a single server with constant service rate V [bps]. When the server turns to the high priority queue, all high priority packets are served before any of those from lower priority classes. The server applies non-preemptive service principle (NPRP), which means that a low-priority packet is not interrupted if a high-priority one comes along while it is under service.

The system has I priority classes and assumes that the class of lower priority index value has higher priority. Each priority class has its own queue of finite length

$b^{(i)}$, $i=1,2,\ldots,I$. Packets in each class are served according to first-come first-served (FCFS) order. Batch of packets of different priority classes arrives to the system according to the Poisson process. Denote by $\lambda^{(i)}$, $i=1,2,\ldots,I$ the incoming traffic intensity of a given class i. The number of packets in each arrived batch follows a discrete random variable X. In general X can be different for each class. In addition, packets of batches of all traffic classes have the same constant size of L [bits].

Since the buffers are finite, in case of overload two cases of rejection rules are analyzed. The first case, when the arriving batch of packets could not fully get into the queue of that certain class the whole batch will be lost, is called complete rejection. Alternatively, the partial rejection could be used, which means if there is no room in the right queue for an arriving batch, the batch will fill the buffer with packets, and the rest will be lost. In the calculation of rejection in this case, the whole batch of packet is considered as lost. An illustration of the considered priority queuing model is shown in *Figure 2*.

5. Analysis of the queuing system

In this section we outline the mathematical analysis of the queuing system presented in Section 4. The proposed model precisely describes the system behaviour without any approximation. Let us see the following systematic steps.

First of all the analysis of the presented queuing system with constant service rate (V) is converted into the simpler problem of I $M_x/G/1/b$ queues. Let us see the system from the ith priority queue point of view. The batch of L-sized packets arrival follows a Poisson process with $\lambda^{(i)}$ parameter, while the queue size is $b^{(i)}$. The service time for a packet in this queue, however, differs from the time needed by the server to serve the packet itself (L/V).

Instead, with the selfish respect to the queue i, the service time begins when the server starts to serve a packet of class i and then finishes when it is ready to serve the next packet of the same queue. It includes the operation time to serve the possible higher priority packets which arrived in the mean time. This service time is denoted by $S^{(i)}$. The basis of our analysis is a recursive calculation of $S^{(i)}$ based on the distribution of the service time of the previous queues and their busy periods $T^{(i)}$.

$$S^{(i+1)} = S^{(i)} + T^{(i)}[N^{(i)}(S^{(i)})], \qquad i = 1,2,\ldots,I.$$

Note that there is another similar recursive formula for the distribution of $T^{(i)}$.

We still need to calculate the distribution of the special service time S^* of the first packet of the batch that arrives to the empty queue. This random variable depends on the state of the other queues. Handling this issue a much more sophisticated recursion is provided.

So we decompose the system into different queues but they are not independent so we encode the dependence of the queues into the service time and special service time. Next, we determine the long-run average distribution of a single $M_x/G/1/b$ queue with special first service time. The probability that there are j packets in the queue, p_j, is defined as the limit of the fraction of time the system spends in state with j packets in the buffer over the operation. To determine this probability we use the theory of regenerative processes. More precisely, we divide the whole service time into the expected values of the time that the queue spends with j packets in the queue.

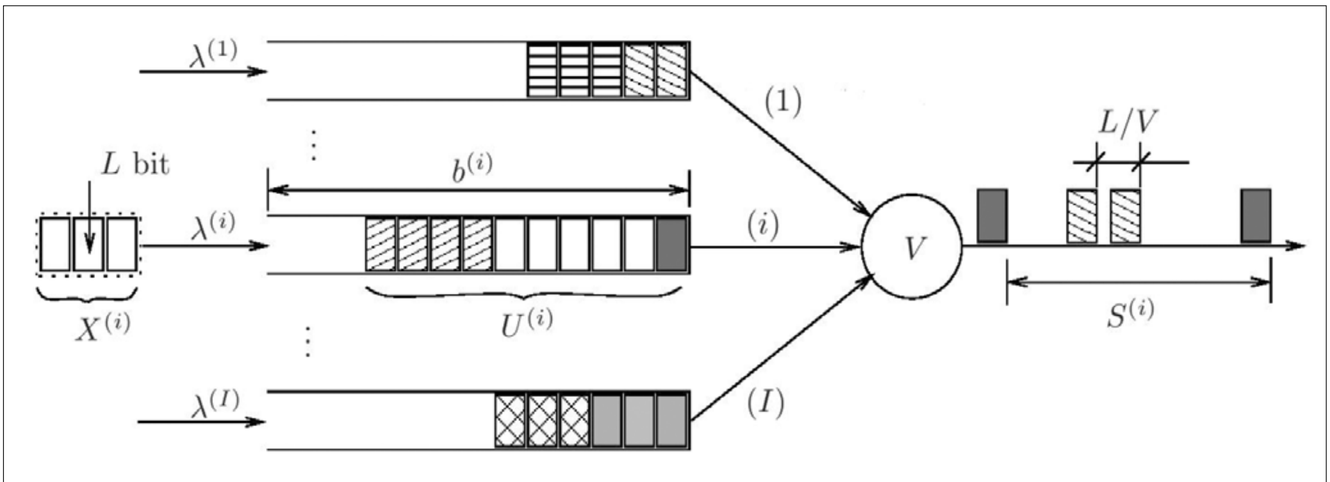
$$p_j = \frac{d_0 t_{0,j} + d_1 t_{1,j} + \cdots d_b t_{b,j} + d_{x_1} t_{x_1,j} + \cdots + d_{x_l} t_{x_l,j}}{d_0 \dfrac{1}{\lambda} + (d_1 + \cdots d_b)\mathbf{E}S + (d_{x_1} + \cdots + d_{x_l})\mathbf{E}S^*}$$

Using the above results the waiting time distribution is given by the following formula:

$$W = R + \sum_{k=1}^{U} S_k + \sum_{k=U+1}^{U+X-1} S_k + \frac{L}{V}.$$

The batch with X packets arrives into the queue during a service time, such that, there are U packets in the queue. The waiting time includes the remaining time R,

Figure 2. The queuing model



that is the time while the service of the first packet in the queue starts, the first sum which is the time is needed for the batch to wait for the service of the packets in the queue, and the second sum regarding as the service times of the packets in the batch itself, except the last one that needs only time of L/V. Since these times are independent simple convolution can be applied. The calculation of the distribution of R is very similar to the calculation of p_j above.

The rejection probability can be easily calculated using the previous paragraphs. Let X be the number of arriving packets in a batch after a sufficiently long time then the probability of the rejection can be formulated into the following form:

$$P_{rej} = \sum \mathrm{P}(U = j)\,\mathrm{P}(X > b - j)$$

Regarding the rejection rules, we have to emphasize two important differences in the analysis. One of them appears in the matrix of the probabilities which tells us how the number of packets changes in the queue when a batch arrives. This matrix is used for the calculation of $S^{(i)}$ and the stationary distribution of the aforementioned Markov chain. The other difference is in $t_{i,j}$ and R. Both of them concern an exponential random variable that describes how long the system has to wait until the change of the queue length if there are j packets in the queue.

The intensity of this time is different for different rejection rules. Namely, if partial rejection is considered then the queue length always changes if a batch arrives while with complete rejection the queue length changes only if the batch fully fits in the buffer.

6. Numerical results

The presented numerical algorithms have been implemented in *C* and for the justification of our analysis and to investigate the different behaviour of complete and partial rejections we provided long run packet-level simulations as well. The system parameters are chosen so that they meet closely the real DSL based access conditions. The load is set to achieve 50%, 70% and 90% utilizations, and the ratios between traffic classes are 4%, 12%, 24%, and 60%, respectively. The packet arrival time is chosen to fit to voice traffic in the first class and to internet-like traffic based on the simple IMIX model of the low-priority classes. Note that the infinite sums and continuous distributions in the numerical calculations are approximated to have the errors less than 10^{-6}.

The tail probability distribution of the service time of class-4 is shown in *Figure 3*. Remind that the service time of a priority packet is the time difference between the service of two consecutive packets in a given priority buffer. The curves show the results of our numerical analysis and simulations of 90% utilization. Simulation results are done for 10^8, 10^7 and 10^6 packet arrivals. It can be observed that the results are almost the same. The only difference between the two curves is that the numerical method can also provide those probabilities where the simulation is less feasible. The other observation is that the service time seems to follow geometrical distribution since the tail distribution is almost a straight line in the log-log scale.

In *Figure 4*, the differences between complete and partial rejections could be seen in terms of the queue length Probability Density Function (PDF) of class-2 under link utilization of 50%. There is a significant difference in the results near the capacity limit of the queue, which cause the difference at the rejection probability.

Our last results in *Figure 5 and 6* show the variance in Cumulative Density Function (CDF) of whole batch of packets waiting times. The difference is not significant compared with the queue length distribution. However, if we rescale the graph to finer the grid we could realize that the probabilities of partial rejection are always below the one of complete rejection.

Figure 3.
The numerical and simulation results of the service time

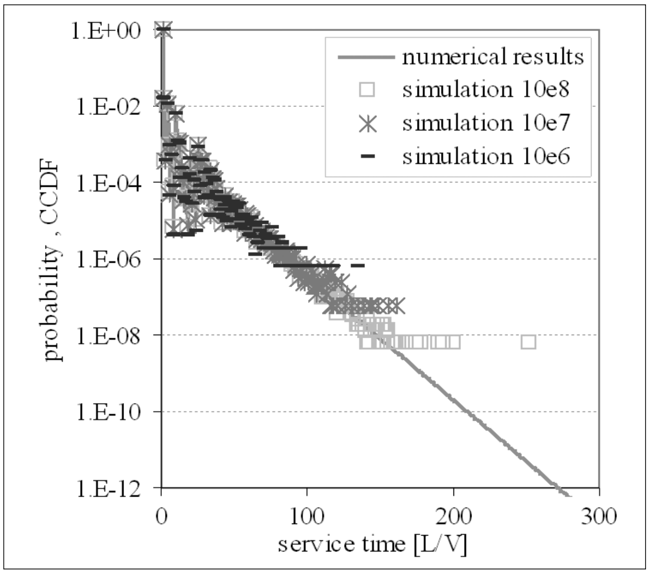
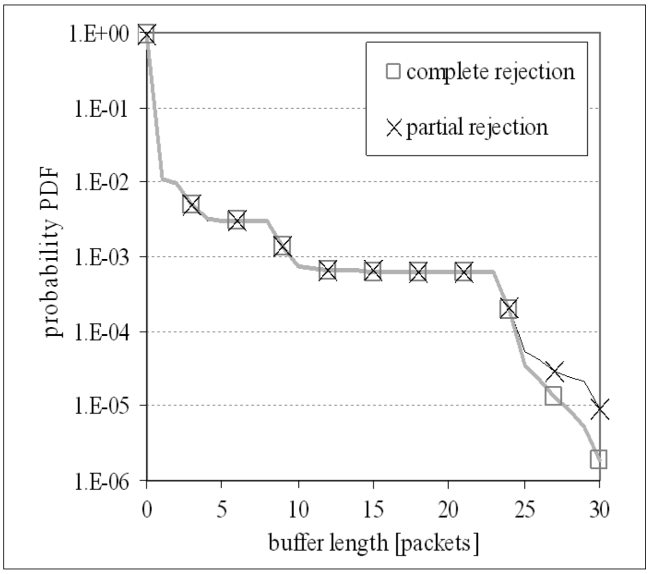


Figure 4.
Queue length distribution for complete and partial rejection rules



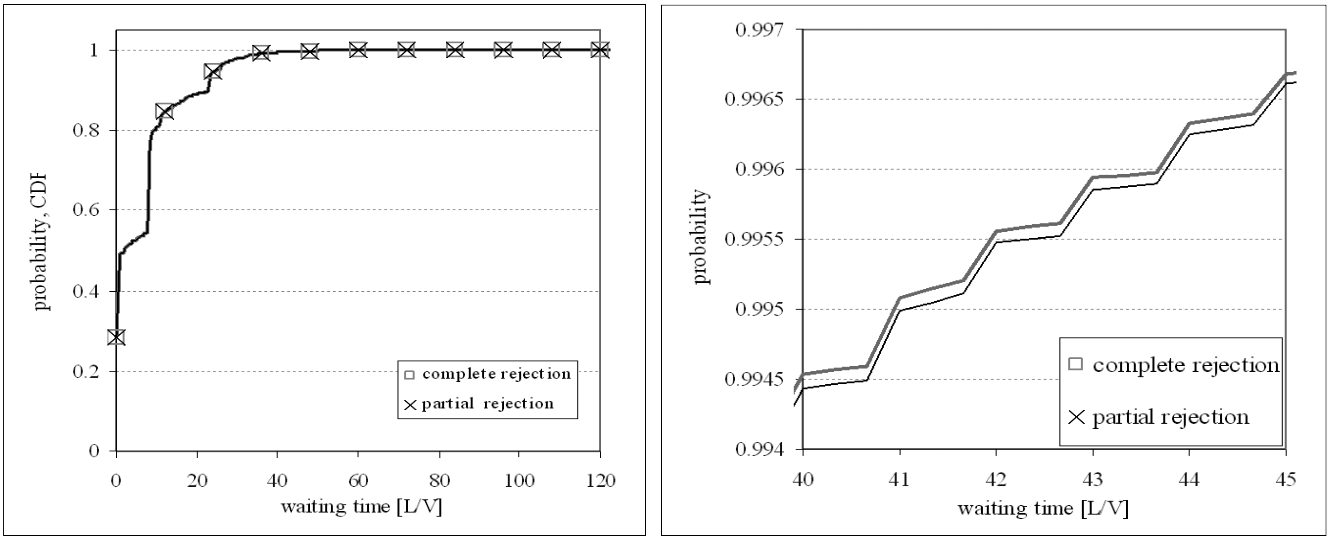


Figure 5-6. Delay distribution in complete and partial rejection rule

7. Conclusions

In this paper an exact analysis of finite buffer priority queuing system with Poisson batch arrivals is provided. We have investigated both the complete and partial rejection rules. The main step of the analysis was the encoding of the dependence structure of the whole system into the service and the special service time in each queue.

The derived results show the practical difference between the partial and the complete rejection: the rejection probabilities are significantly different while the delays almost equal. Besides, the feasibility of the numerical analysis model has been proven, comparing it with long-term simulation results.

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