Short impulse propagation in waveguides

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One of the most important research topics is the investigation of (short) impulse propagation in waveguides. The known solutions are based upon the well-known monochromatic approaches, examining the different frequencies separately or building the model and the theory on a fundamentally monochromatic starting point (e.g. permittivity tensor, which is defined originally by assuming an type solution form). In this paper a completely new theoretical model and solving method will be presented for a rectangular waveguide filled by vacuum, excited by an arbitrarily formed electromagnetic signal (Dirac or real, even short impulse).

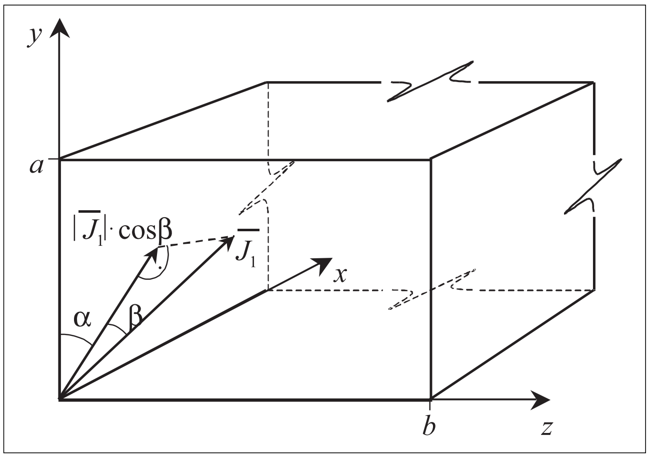
This method avoids the application of the former assumptions regarding the sinusoidal waveforms.

The obtained closed-formed solution leads back to the former ones known for monochromatic excitation, by using a sinusoidal excitation with a given frequency, but obviously the new formula results a general solution of the problem.

Introduction

The model, in which the new theoretical method will be presented, is a rectangular waveguide filled by vacuum and bordered by perfect conductor walls.

Fig.1. The structure of the model



The generally directed exciting current density is

$$\bar{J}_1 = J_{1x}\cdot\bar{i} + J_{1y}\cdot\bar{j} + J_{1z}\cdot\bar{k}$$
$$\left|\bar{J}_1\right| = \delta(t)\cdot\delta(x)\cdot B_1(y)\cdot B_2(z) \qquad (1)$$

where $B_1(x)$ and $B_2(z)$ are envelope functions containing the boundary conditions

$$B_1(0) = B_1(a) \equiv 0 \quad \text{and} \quad B_2(0) = B_2(b) \equiv 0 \qquad (2)$$

So the current density is (3)

$$\bar{J}_1 = \left|\bar{J}_1\right|\sin\beta\,\bar{i} + \left|\bar{J}_1\right|\cos\beta\,\cos\alpha\,\bar{j} + \left|\bar{J}_1\right|\cos\beta\,\sin\alpha\,\bar{k}$$

Further – without any theoretical restriction – let the following, sufficiently general form of the excitation be applied (4):

$$\bar{J}_{1x} \equiv 0$$
$$\bar{J}_1 = \delta(t)\delta(x)B_1(y)B_2(z)\cos\alpha\,\bar{j} + \delta(t)\delta(x)B_1(y)B_2(z)\sin\alpha\,\bar{k}$$
$$\left|\bar{J}_1\right| = J_1 = \delta(t)\delta(x)B_1(y)B_2(z)$$

The new solving method

The theoretical basis of the solving method can be found in [1, 2] for transient plane waves.

The equations to be solved are Maxwell' equations [3, 8]

$$I.\quad \nabla\times\bar{H} = \bar{J}_1 + \varepsilon_0\frac{\partial\bar{E}}{\partial t} \qquad (5)$$
$$II.\quad \nabla\times\bar{E} = -\mu_0\frac{\partial\bar{H}}{\partial t}$$
$$III.\ \nabla\bar{H} = 0$$
$$IV.\ \nabla\bar{E} = \frac{\rho}{\varepsilon_0}$$

Let the retarded potential be introduced on the well known way

$$\nabla\times\bar{A} = \bar{H}$$
$$\bar{E} + \mu_0\frac{\partial\bar{A}}{\partial t} = -\nabla\psi \qquad (6)$$

The Lorenz-condition is valid, as usual

$$\left(\nabla\bar{A} + \varepsilon_0\frac{\partial\psi}{\partial t}\right) = 0 \qquad (7)$$

So, the equation to be solved is

$$\nabla^2 \overline{A} - \varepsilon_0 \mu_0 \frac{\partial^2 \overline{A}}{\partial t^2} = -\overline{J}_1 \tag{8}$$

As the excitation is a generally formed signal with exact starting point according to time and space, the Laplace-transformation can be applied, as

$$\begin{aligned} t &\stackrel{L}{\longleftrightarrow} s \\ x &\stackrel{L}{\longleftrightarrow} p \\ y &\stackrel{L}{\longleftrightarrow} u \\ z &\stackrel{L}{\longleftrightarrow} l \\ f(t,x,y,z) &\stackrel{L}{\longleftrightarrow} F(s,p,u,l) \end{aligned} \tag{9}$$

Because of the presence of derivative terms, initial values according to all coordinates will appear. Usually these initial values contain information regarding the energetic state of the medium. However, in this case the medium is considerable to be free of energy before the switching on of the excitation. Therefore in the further all the initial values have to be taken into account as 0.

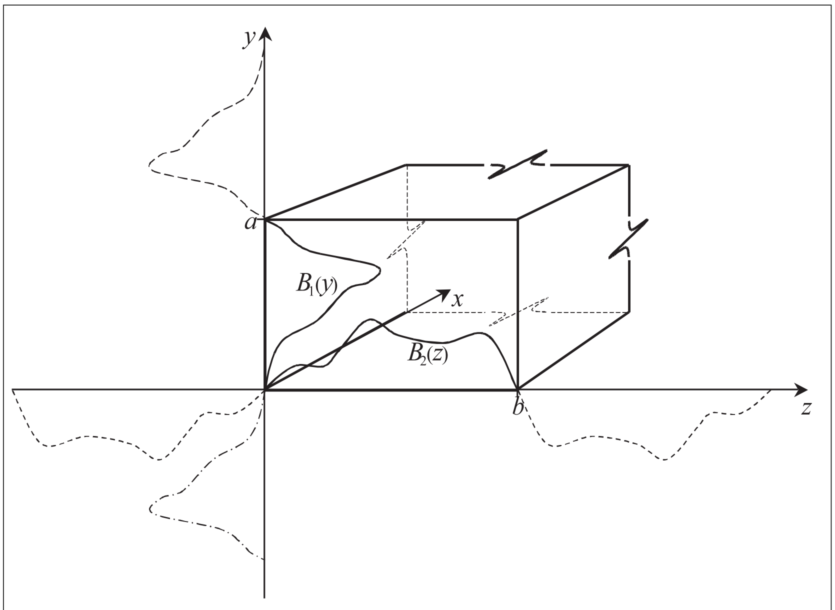
The transformed equations to be solved are:

$$\begin{aligned} H_x(s,p,u,l) &= u\,A_z(s,p,u,l) - l\,A_y(s,p,u,l) \\ H_y(s,p,u,l) &= -p\,A_z(s,p,u,l) \\ H_z(s,p,u,l) &= p\,A_y(s,p,u,l) \end{aligned} \tag{10}$$

further

$$\begin{aligned} E_x(s,p,u,l) &= \frac{1}{\varepsilon_0}\frac{1}{s}\left[p\,u\,A_y(s,p,u,l) + p\,l\,A_z(s,p,u,l)\right] \\ E_y(s,p,u,l) &= \frac{1}{\varepsilon_0}\frac{1}{s}\left[u^2 A_y(s,p,u,l) + u\,l\,A_z(s,p,u,l)\right] - \mu_0\,s\,A_y(s,p,u,l) \\ E_z(s,p,u,l) &= \frac{1}{\varepsilon_0}\frac{1}{s}\left[u\,l\,A_y(s,p,u,l) + l^2 A_z(s,p,u,l)\right] - \mu_0\,s\,A_z(s,p,u,l) \end{aligned} \tag{11}$$

Fig.2. The envelope functions



The Laplace-transformed form of the exciting current density is

$$\begin{aligned} J_1(s,p,u,l) &= \\ &= \iiiint_0^\infty \delta(t)\delta(x) B_1(y) B_2(z) \cdot e^{-st} \cdot e^{-px} \cdot e^{-uy} \cdot e^{-lz} \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &= 1 \cdot 1 \cdot B_1(u) B_2(l) \end{aligned} \tag{12}$$

It is very important to choose B_1 and B_2 functions suitably. These envelope functions contain the boundary conditions resulted from the geometrical structure of the model. This can be seen in *Fig.2.*

The boundary conditions to be validated are

$$B_1(0) = B_1(a) = B_2(0) = B_2(b) \equiv 0 \tag{13}$$

As it is usual [3], the envelope functions B_1 and B_2 can be extended periodically, and it is possible to describe them by Fourier-series:

$$\begin{aligned} B_1(y) &= \sum_{m=0}^{\infty} C_m \cdot \mathrm{e}^{\mathrm{j}m\frac{\pi}{a}y} \\ C_m &= \frac{1}{2a}\int_{-a}^{a} B_1(y) \cdot \mathrm{e}^{-\mathrm{j}m\frac{\pi}{a}y}\,\mathrm{d}y \end{aligned} \tag{14}$$

and

$$\begin{aligned} B_2(z) &= \sum_{n=0}^{\infty} C_n \cdot \mathrm{e}^{\mathrm{j}n\frac{\pi}{b}z} \\ C_n &= \frac{1}{2b}\int_{-b}^{b} B_2(z) \cdot \mathrm{e}^{-\mathrm{j}n\frac{\pi}{b}z}\,\mathrm{d}z \end{aligned} \tag{15}$$

where C_m and C_n are Fourier-coefficients, a and b are geometrical parameters of the waveguide, m and n are integers

$$\begin{aligned} m &= 0, \pm 1, \pm 2, \ldots \\ n &= 0, \pm 1, \pm 2, \ldots \end{aligned} \tag{16}$$

The Laplace-transformed forms of (19) and (20) are

$$\begin{aligned} B_1(u) &= \sum_{m=-\infty}^{\infty} \frac{C_m}{u - \mathrm{j}m\frac{\pi}{a}} \\ B_2(l) &= \sum_{n=-\infty}^{\infty} \frac{C_n}{l - \mathrm{j}n\frac{\pi}{b}} \end{aligned} \tag{17}$$

The poles according to p can be determined from

$$p^2 + u^2 + l^2 - \varepsilon_0 \mu_0 s^2 = 0 \tag{18}$$

Investigating the poles according to u and l it becomes obvious, that only the poles originating from the excitation will yield terms different from zero in the amplitudes. Executing the well known steps of the inverse Laplace-transformation on the common way and using the

$$s = \mathrm{j}\omega \tag{19}$$

substitution, the spectral forms of the filed components depending on the spatial variables are:

$$H_x(\omega,x,y,z)=\sum_m\sum_n \frac{P_-C_mC_n}{2k_x(\omega)}\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}-\sum_m\sum_n \frac{P_-C_mC_n}{2k_x(\omega)}\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]}$$

$$H_y(\omega,x,y,z)=\sum_m\sum_n C_mC_n\sin\alpha\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}+\sum_m\sum_n C_mC_n\sin\alpha\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]} \tag{20}$$

$$H_z(\omega,x,y,z)=\sum_m\sum_n C_mC_n\cos\alpha\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}+\sum_m\sum_n C_mC_n\cos\alpha\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]}$$

and

$$E_x(\omega,x,y,z)=\sum_m\sum_n \frac{-P_+C_mC_n}{2\omega\varepsilon_0}\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}+\sum_m\sum_n \frac{P_+C_mC_n}{(-2)\omega\varepsilon_0}\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]}$$

$$E_y(\omega,x,y,z)=\sum_m\sum_n \frac{\left[(-jm\pi/a)(jP_+)-\varepsilon_0\mu_0\omega^2\cos\alpha\right]C_mC_n}{2j\varepsilon_0\omega\, jk_x(\omega)}\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}+$$
$$+\sum_m\sum_n \frac{\left[(-jm\pi/a)(jP_+)-\varepsilon_0\mu_0\omega^2\cos\alpha\right]C_mC_n}{(-2)j\varepsilon_0\omega\, jk_x(\omega)}\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]} \tag{21}$$

$$E_z(\omega,x,y,z)=\sum_m\sum_n \frac{\left[(-jn\pi/b)(jP_+)-\varepsilon_0\mu_0\omega^2\sin\alpha\right]C_mC_n}{2j\varepsilon_0\omega\, jk_x(\omega)}\cdot e^{j[k_x(\omega)\cdot x\cdot M\cdot N]}+$$
$$+\sum_m\sum_n \frac{\left[(-n\pi/b)P_+-\varepsilon_0\mu_0\omega^2\sin\alpha\right]C_mC_n}{(-2)j\varepsilon_0\omega\, jk_x(\omega)}\cdot e^{j[-k_x(\omega)\cdot x\cdot M\cdot N]}$$

where

$$P_+=m\frac{\pi}{a}\cos\alpha+n\frac{\pi}{b}\sin\alpha \qquad P_-=-m\frac{\pi}{a}\cos\alpha+n\frac{\pi}{b}\sin\alpha \qquad M=m\frac{\pi}{a}y \qquad N=n\frac{\pi}{b}z$$

$$k_x(\omega)=\sqrt{\varepsilon_0\mu_0\omega^2-\left(m\frac{\pi}{a}\right)^2-\left(n\frac{\pi}{b}\right)^2}.$$

It can be seen, that one term in the field components propagates forward, the other propagates backward, considering the location of the excitation as a starting point ($x=0$) in the assumed infinitely long wave-guide.

The limiting wave-length (and the limiting frequency, respectively) can be obtained from (20) and (21) as

$$\varepsilon_0\mu_0\omega^2-\left(m\frac{\pi}{a}\right)^2-\left(n\frac{\pi}{b}\right)^2=0 \qquad \lambda_{m,n}=\frac{2ab}{\sqrt{(mb)^2+(na)^2}} \tag{22}$$

By the application of the formal inverse Fourier-transformation, the complete time-space dependent, exact form of the propagating electric and magnetic field-components can be obtained as

$$H_x(t,x,y,z)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n\frac{P_-C_mC_n}{k_x(\omega)}\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}-\sum_m\sum_n\frac{P_-C_mC_n}{k_x(\omega)}\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega$$

$$H_y(t,x,y,z)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n C_mC_n\sin\alpha\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}+\sum_m\sum_n C_mC_n\sin\alpha\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega$$

$$H_z(t,x,y,z)=\frac{-1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n C_mC_n\cos\alpha\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}+\sum_m\sum_n C_mC_n\cos\alpha\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega$$

$$E_x(t,x,y,z)=\frac{-1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n\frac{P_+C_mC_n}{\omega\varepsilon_0}\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}+\sum_m\sum_n\frac{P_+C_mC_n}{\omega\varepsilon_0}\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega$$

$$E_y(t,x,y,z)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n\frac{\left[(-m\pi/a)P_+-\varepsilon_0\mu_0\omega^2\cos\alpha\right]C_mC_n}{\varepsilon_0\omega\, k_x(\omega)}\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}-\right.$$
$$\left.-\sum_m\sum_n\frac{\left[(-m\pi/a)P_+-\varepsilon_0\mu_0\omega^2\cos\alpha\right]C_mC_n}{\varepsilon_0\omega\, k_x(\omega)}\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega \tag{23}$$

$$E_z(t,x,y,z)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\left\{\sum_m\sum_n\frac{\left[(-n\pi/b)P_+-\varepsilon_0\mu_0\omega^2\sin\alpha\right]C_mC_n}{\varepsilon_0\omega\,k_x(\omega)}\cdot e^{jT_+}\cdot e^{jM}\cdot e^{jN}-\right.$$
$$\left.-\sum_m\sum_n\frac{\left[(-n\pi/b)P_+-\varepsilon_0\mu_0\omega^2\sin\alpha\right]C_mC_n}{\varepsilon_0\omega\,k_x(\omega)}\cdot e^{jT_-}\cdot e^{jM}\cdot e^{jN}\right\}d\omega \quad (23)$$

where

$$T_+=\omega t+k_x(\omega)\cdot x \qquad T_-=\omega t-k_x(\omega)\cdot x$$

Numerical results

As a first application of the obtained closed-formed solution, let an Earth-ionosphere wave-guide model be calculated.

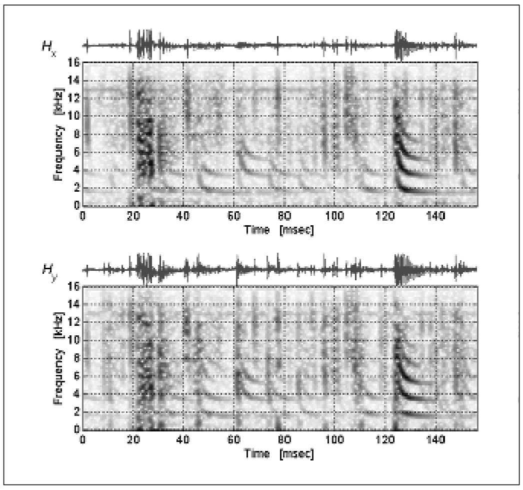
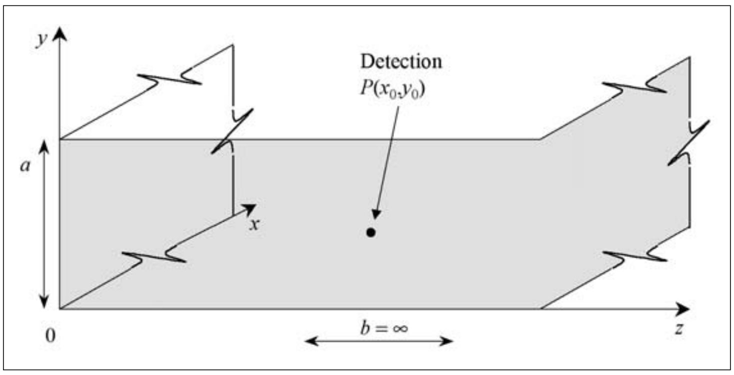


Fig.3. Observed dynamic spectra and time functions for two magnetic field-components (Marion Island, 2001.04.22. 02:30:07)

In the model, let $b=\infty$ be assumed (wave propagation between two infinite perfect conductor plain walls), the excitation is in the $x=0$ plane. It is shown in *Fig.4.*

Fig.3. shows observed dynamic spectra detected on the terrestrial surface, propagated in the Earth-ionosphere layer. Many papers investigate the propagation of these signals, and it is known that the form of the dynamic spectrum is caused by the presence of the Earth-ionosphere waveguide [4, 5, 6, and 7].

Fig.4/a.



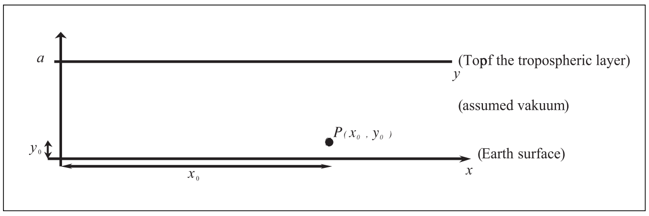


Fig.4/b. The model of the Earth-ionosphere wave-guide

But the theoretical description of the problem is based on the monochromatic wave-propagation of guided waves [8].

In the model-calculation the height of the bottom of the ionosphere is at 85 km, 2000 km is the traveled propagation path, 1000 m is the height of the antenna (location of detection). The excitation is a Dirac and all the Fourier coefficients are taken into consideration with 1 value, $\alpha=45°$, and $m=0,\ldots,5$ modes are taken into consideration (but all these parameters are flexible and modifiable).

The calculated time functions and dynamic spectra of H_x and H_y at the P point is shown in *Fig.5.*

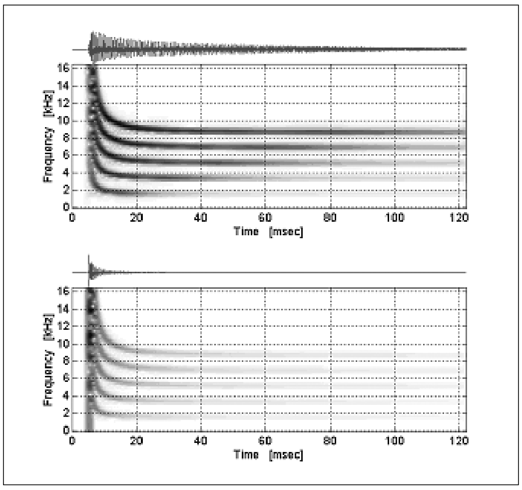


Fig.5. Calculated time functions and dynamic spectra of field components for Dirac excitation

In the comparison of Fig.3. and 5. some similarities are recognizable. The real impulse behavior can well explain the parallel traces in the spectra. The number of the branches depends on the number of Fourier-coefficients (modes) taken into consideration. Moreover, the distance among the branches depends on the height of the waveguide, or with other words, the thickness of the troposphere, the actual height of the bottom of the ionosphere (the height of the D layer).

Conclusions

In this paper a closed-formed solution is presented for guided waves excited by real transient signals.

The presented solving method is general and fully analytical. The excitation is a real arbitrarily shaped transient signal. Applying the presented solving method for Dirac excitation, the transfer function of the waveguide can be described. This result opens the way for investigation of real (short) impulse propagation in waveguides and the transient propagation phenomena. The geometrical structure can be developed further.

The closed-formed solution is well applicable for computers in order to calculate numerical results of the analytical solutions.

As a possible application of the new result in a special case, a geophysical example was presented. From the comparison of Fig.3. and Fig.5. it can be seen, that the model describes well some observed phenomena. By the application of this solution it became possible to monitor the bottom of the ionosphere continuously. It is not necessary to use the averaging of many observed spherics for the estimation of the height of the D layer, because the more exact new model makes it possible to use each individual observed spheric separately.

Moreover, the distance of the lightning source can be estimated from the dispersion fitting of the measured and calculated signals. The direction of incidence is well determinable from the ratio of H_x and H_y components.

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